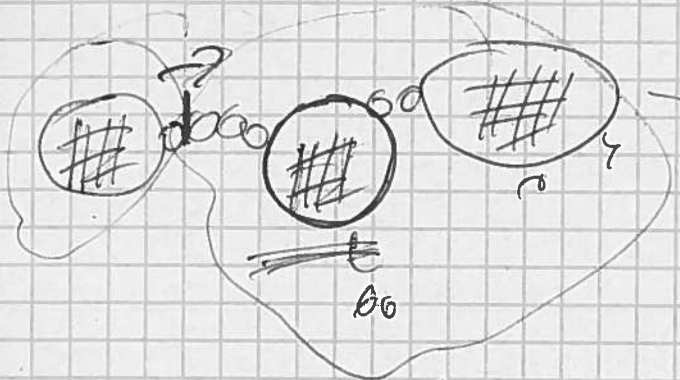
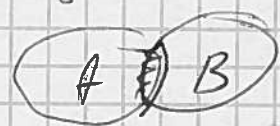


(A,B)

$G = A \cup B$

$E(A) \cap E(B) = \emptyset$



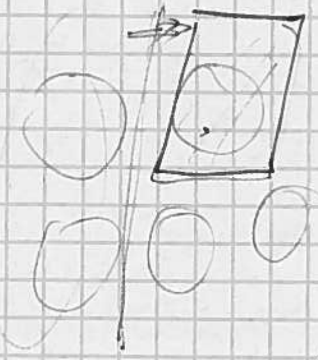
tangle $\mathcal{T}$: množina separací
řád Θ

velikost $= |V(A \cap B)|$

① Jestliže (A,B) je separace velikosti $< \Theta$,
pak $(A,B) \in \mathcal{T}$, nebo $(B,A) \in \mathcal{T}$

② $(A_1,B_1), (A_2,B_2), (A_3,B_3) \in \mathcal{T}$
$\Rightarrow A_1 \cup A_2 \cup A_3 \neq G$

③† $(A,B) \in \mathcal{T} \Rightarrow V(A) \neq V(G)$



(A,B), (B,A), (B,A)

$(A,B) \in \mathcal{T} \Rightarrow (B,A) \notin \mathcal{T}$

$(A_1,B_1) \in \mathcal{T}$, $(A_2,B_2) \in \mathcal{T}$,

$(A_1 \cup A_2, B_1 \cap B_2)$ má velikost $< \Theta$

$\Rightarrow (A_1 \cup A_2, B_1 \cap B_2) \in \mathcal{T}$

$(B_1 \cap B_2, A_1 \cup A_2) \in \mathcal{T}$

$$A_1 \cup A_2 \cup (B_1 \cap B_2) = G \quad \lightning$$

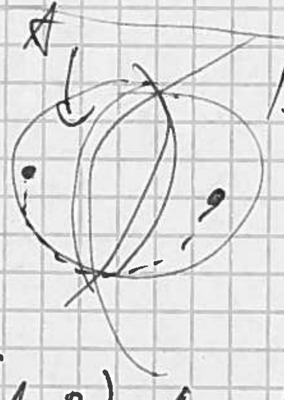
jen pro $\Theta \geq 3$:

$$3 \Leftarrow \forall e \; (K_e, G \setminus e) \in \mathcal{T}$$

$$(G \setminus e, K_e) \notin \mathcal{T} \rightsquigarrow (K_e, G \setminus e) \in \mathcal{T}$$

$\vdash$ sporem $(A, B) \in \mathcal{T}$, $V(A) = V(G)$

K_n



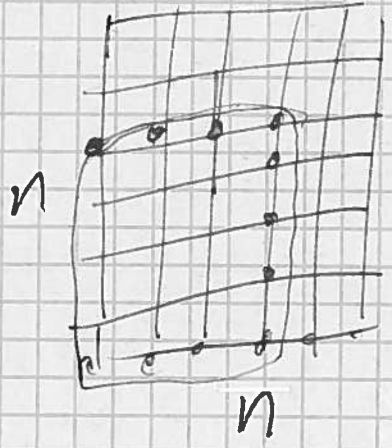
$$\mathcal{T} = \left\{ (A, B) : |V(A)| < \frac{2}{3} n \right\}$$

řádo $\lceil \frac{2}{3} n \rceil$

$v \notin A_1 \dots > \frac{1}{3} n$

$v \notin A_2 \quad > \frac{1}{3} n$

$A_3 \quad > \frac{1}{3} n$



X ... množina hran

∂X ... vrcholy sousedící s X i $E \setminus X$

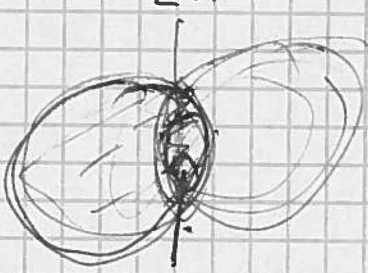
X malé

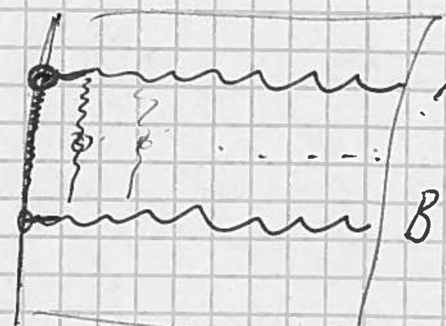
$\boxed{|\partial X| < n}$ a X neobsahuje hrany celého řádku

velikosti $< n$

$\mathcal{T} = \{(A,B) : E(A) \text{ je malé}\}$ řádu n

$< n$

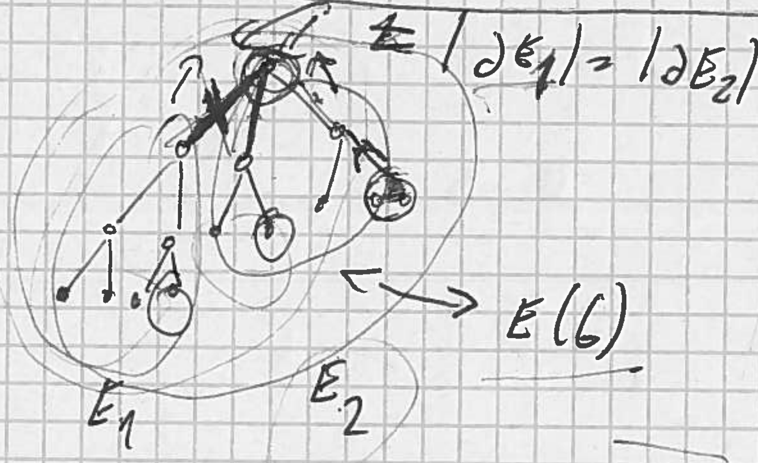


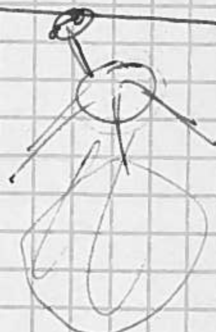


[X_1, X_2, X_3 malé $\Rightarrow X_1 \cup X_2 \cup X_3 \neq E(G)$

---

bw:





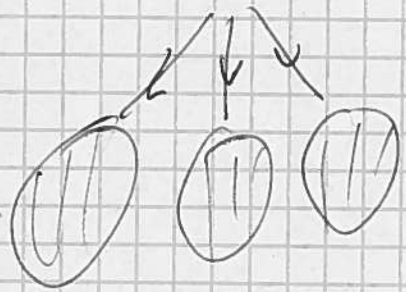
bw = min šířka branch dek. G.

$bw \leq \boxed{tw+1} \leq \boxed{\frac{3}{2}} bw$

$bw(G) =$ řád největší tangle v G

$\exists$ tangle ř. $\theta \Rightarrow bw(G) \geq \theta$

$(G \setminus e, K_e) \in \mathcal{T}$ ↯ 3



↯ 2

$\theta \leq bw \leq tw + 1 \leq \boxed{4\,\theta(G) + \cancel{1}\,4}$

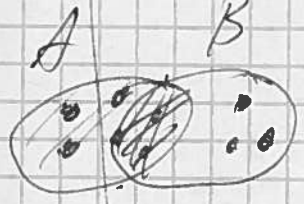
$\leq \frac{3}{2} bw = \frac{3}{2}\,\theta(G)$

G nemá tangli ř. Θ

$\Rightarrow$ G má dekomp.

sbr. vel $\leq 4\Theta - 3$

Lemma: $Z \subseteq V(G)$, $|Z| = 3\Theta - 2$



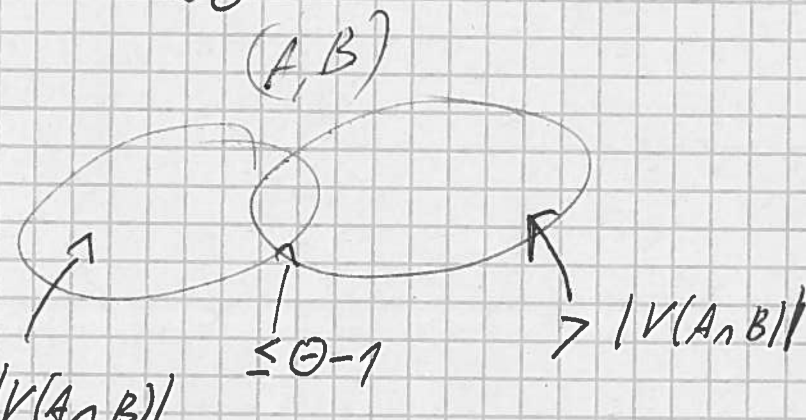
Nechť $\forall\, (A,B)$ velikosti $< \Theta$

je $|(Z \cup V(A)) \cap V(B)| \geq 3\Theta - 2$ nebo

$|(Z \cup V(B)) \cap V(A)| \geq 3\Theta - 2$

$\Rightarrow$ G obsahuje tangli řádu Θ.

$T = \{ \underset{\text{vel.} < \Theta}{(A,B)} : |Z \cup V(A)| \leq |V(A \cap B)| \}$



$|(Z \cup V(A)) \cap V(B)| = |V(A \cap B)| + |Z \setminus V(A)|$

$< |Z \cap V(A)| + |Z \setminus V(A)|$

$= |Z| = 3\Theta - 2$

$$|Z \cap V(A_1)| \le |V(A_1 \cap B_1)| \le \Theta - 1$$

$$|Z \cap V(A_2)|$$

$$|Z \cap V(A_3)|$$

$$|Z \cap V(A_1 \cup A_2 \cup A_3)| \le 3\Theta - 3 < |Z|$$

$$\neq G$$

$Z \subseteq V(G)$: $|Z| \le 3\Theta - 2$

$\Rightarrow \exists$ str. dekomp. s br. vel $\le 4\Theta - 3$,

kde $Z \subseteq$ brambory kořene

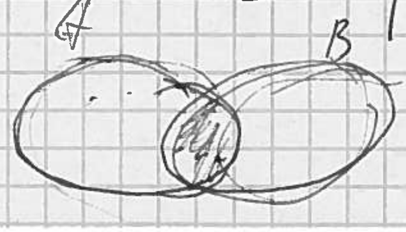
Jestliže $|V(G)| < 3\Theta - 2$ $\rightsquigarrow$ 1 brambora

Jinak $|V(G)| \ge 3\Theta - 2$, búno $|Z| = 3\Theta - 2$

dle Lemma: $\exists$ (A,B) vel. Θ,

$|(Z \cup V(A)) \cap V(B)| < 3\Theta - 2$

& $|(Z \cup V(B)) \cap V(A)| < 3\Theta - 2$.

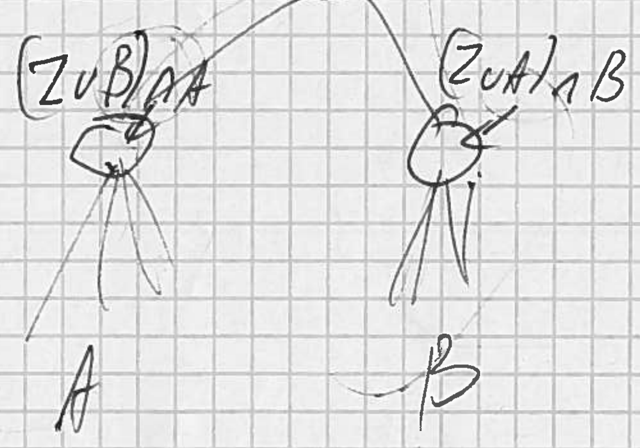


indukce A, $(Z \cup V(B)) \cap V(A)$

B, $(Z \cup V(A)) \cap V(B)$

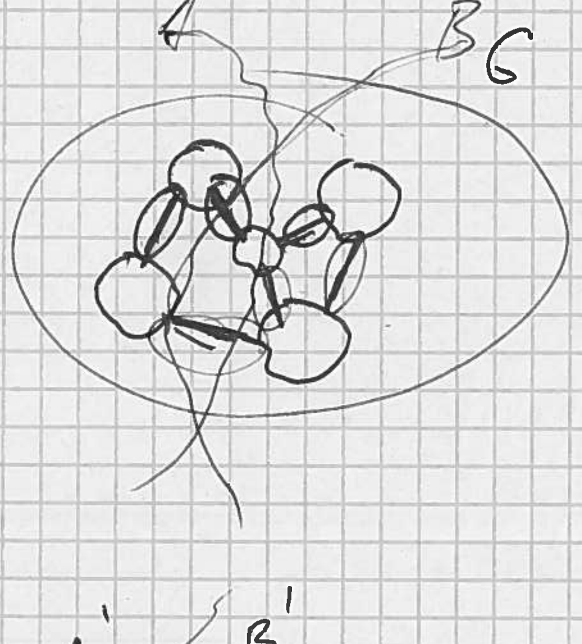
$Z \cup (A \cap B) \leftarrow |\cdot| \le (3\theta - 2) + (\theta - 1)$

$= 4\theta - 3$



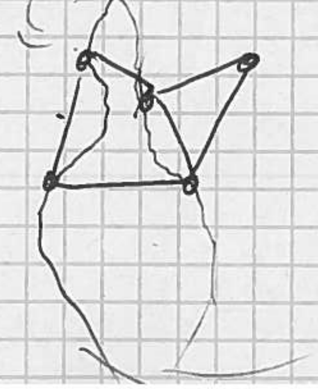
$$H \preceq_m G$$

$$\mathcal{T}_H \text{ řádu } \theta \rightsquigarrow \mathcal{T} \text{ řádu } \theta$$



vel $< \theta$

$$\mathcal{T} = \Big\{ (A,B) : \exists (A',B') \text{ sep.} \in \mathcal{T}_H \text{ t.ž. } E(A') = E(A) \cap E(H),\ E(B') = E(B) \cap E(H) \Big\}$$



$X \subseteq V(G) \qquad |X| < \Theta$

$\mathcal{T}$ tangle řádu Θ v G

$\Downarrow$

$\mathcal{T}-X$ tangle řádu $\Theta - |X|$ v $G-X$

$$\mathcal{T}-X = \left\{ (A', B') : \begin{array}{l} \exists (A,B) \in \mathcal{T} \\ \text{t.ž. } X \subseteq V(A \cap B) \\ A' = A \setminus X \\ B' = B - X \end{array} \right\}$$

$\mathcal{T}$ řádu Θ, $X \subseteq V(G)$: (X, G)

$r(X)$ = nejmenší velikost $(A,B) \in \mathcal{T}$

$X \subseteq V(A)$

neexistuje-li, pak Θ

(i) $0 \le r(X) \le |X|$

(ii) $X \subseteq Y \Rightarrow r(X) \le r(Y)$

(iii) $r(X \cap Y) + r(X \cup Y) \le r(X) + r(Y)$

nezávislé množiny: $|X| = r(X) \Rightarrow |X| \le \Theta$ a neexistuje $(A,B) \in \mathcal{T}$ velikost $< |X|$ s $X \subseteq V(A)$