

5TH TUTORIAL ON RANDOMIZED ALGORITHMS

Eigenvalues, expanders, and distances

1. Compute the eigenvalues and eigenvectors of the following graphs:

- a) K_n , the complete graph on n vertices.
- b) $K_{n,n}$, the complete bipartite graph with partites of size n each.
- c) *Bonus*: C_n , the cycle on n vertices.

2. Let A, B be two disjoint sets of vertices where $|A| = |B| = n$. For a fixed $d \geq 5$, we choose d uniformly at random edges from each vertex from A to B . We show that with constant positive probability each set $S \subseteq A$ of size $|S| \leq n/d$ has more than $d|S|/4$ neighbors.