4^{TH} TUTORIAL ON RANDOMIZED ALGORITHMS

Eigenvalues and Chernoff bounds

1. Show that a connected d-regular graph is bipartite iff the least eigenvalue of its adjacency matrix is $-d$.

2. Give an example of n *dependent* random variables $X_i \in \{0,1\}, i = 1, \dots, n$ such that Chernoff bounds do not hold for $\sum_i X_i$.

3. Propose an algorithm to generate a random bipartite d-regular graph and argue that it is efficient.

4. *Distinguishing coins.* You are given two coins. One is fair and the other one has $\Pr[\text{tails}] = 1/4$. We use the following algorithm to distinguish those:

- Pick a coin and toss it n times.
- Let $\hat{p}$ be the probability of getting a tails (number of tails over n).
- If $\hat{p} \geq 3/8$ we say this coin is fair.

Show that if $n \geq 32 \ln(2/\delta)$ then our algorithm answers correctly with probability at least $1 - \delta$.

5. *Simulating a fair coin using a biased coin and vice versa.*

a) We are given a fair coin $\Pr[tails] = 0.5$. Show how to generate a random bit with $\Pr[1] = p$ for a given $p \in (0,1)$ (both $p = 0$ and $p = 1$ are a bit boring).
   i) First, assume that $p = k/2^\ell$ for some integers k and ℓ.
   ii) What about any rational p? What about irrational p?

b) We are given a biased coin – we do not even know $p = \Pr[tails]$. We are sure that $\Pr[tails] \in (0,1)$. Generate a fair coin toss.