NDMI025 – Randomized Algorithms (Pravděpodobnostní algoritmy) LS 2023 – Jiří Sgall

Tutorial 2 – Feb 21

(1) We are collectors and we want to collect all n kinds of coupons. Coupons are sold in packages which all look the same. Thus when we buy an coupon, we buy one of n kinds uniformly at random. This is known as the *coupon collector* problem.

1. What is the expected number of coupons we need to buy to get all kinds?
2. How many coupons do we need to buy to have probability at least $1 - q$ of collecting all kinds?
3. What is the Markov chain? Is this similar to a random walk on some graph?

(2) A cat and a mouse each independently take a random walk on a cycle of length $2n+1$. (In each step, they both take a random step, i.e., both random walks are synchronized.) The game ends (the cat eats the mouse) if they land at the same vertex at the same time. (If they traverse the same edge in the opposite directions, nothing happens.) Find the expected number of steps of the game if the cat and the mouse start (a) at adjacent nodes and (b) at distance n (i.e., at the largest possible distance).

(3)

1. Find a family of oriented graphs with as large hitting time as possible.
2. What if we additionally require that they have constant in-degree and constant out-degree?

(4)

1. What is the hitting time on a complete graph?
2. A lollipop is an undirected graph on n vertices consisting of a complete graph on t vertices and a path on $s = n - t$ additional vertices, connected by an edge from one endpoint to one of the vertices of the complete graph. Let u be some other vertex of the complete graph (i.e., some vertex of degree $t - 1$) and v be the other endpoint of the path (i.e., the single vertex of degree 1). Consider the random walk on this graph. Compute the hitting time h_{uv}, i.e., the expected time of reaching v from u, as well as h_{vu}, at least asymptotically.
3. Determine also the values of t for which the hitting times are maximized or minimized.