Streaming Algorithms
for
Bin Packing and Vector Scheduling

Graham Cormode and Pavel Veselý
University of Warwick

WAOA 2019, Munich

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First WAOA talk containing “streaming” . . .

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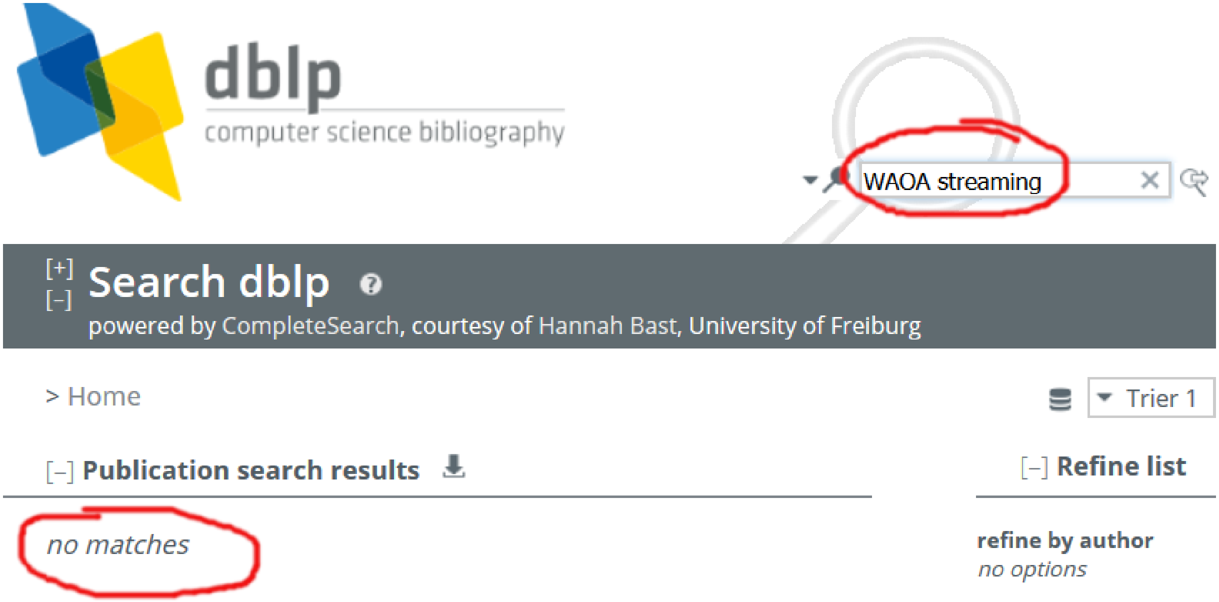

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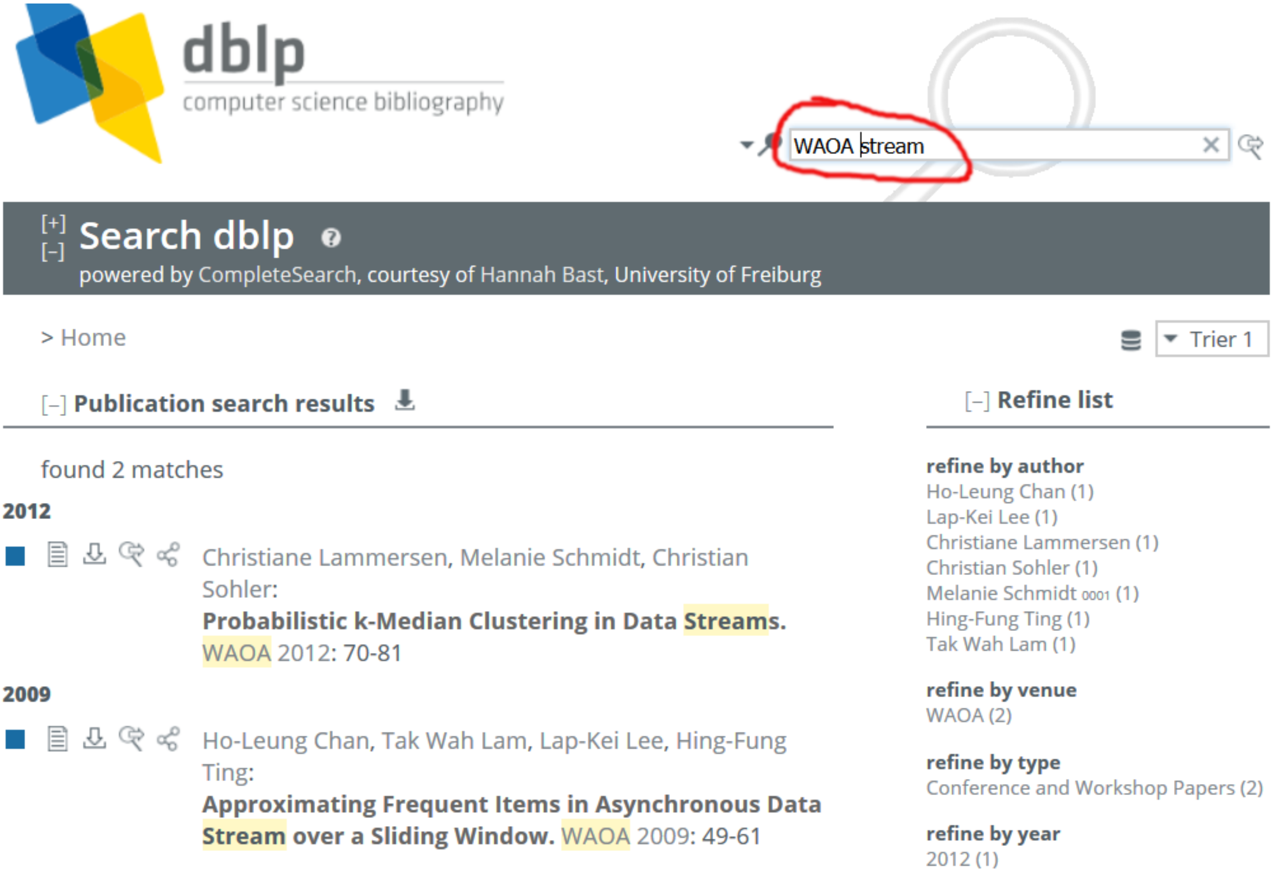
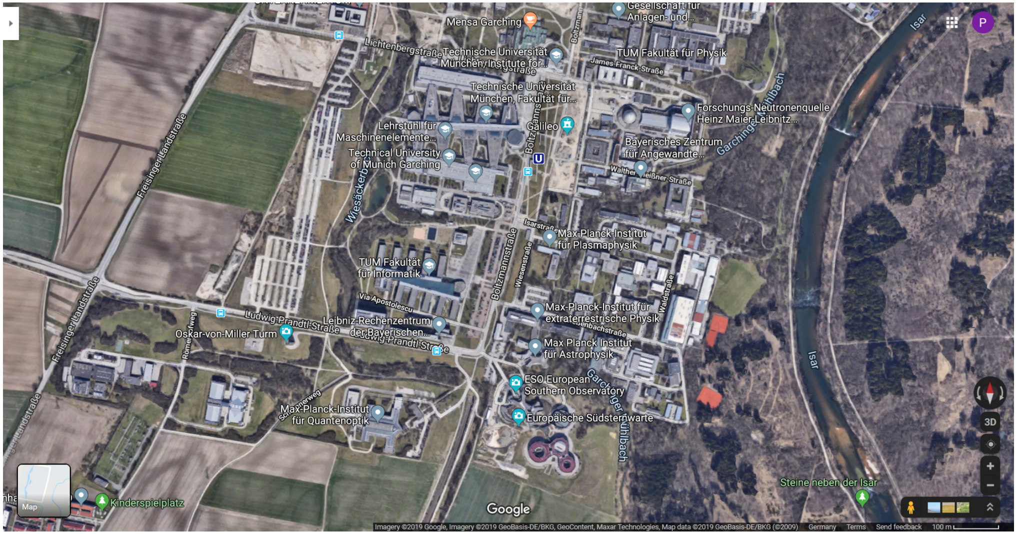
. . . but not the first one on "data streams"



Overview


Mensa Garching
Gesellschaft für Anlagen- und...
Lichtenbergstraße
Technische Universität München, Institute for...
TUM Fakultät für Physik
James-Franck-Straße
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Lehrstuhl für Maschinenelemente...
Galileo
Technical University of Munich Garching
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Freisinger Landstraße
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Ludwig-Prandtl-Straße
Leibniz-Rechenzentrum der Bayerischen...
Oskar-von-Miller Turm
Max Planck Institut für Astrophysik
ESO European Southern Observatory
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Isar
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Kinderspielplatz
Map
Google
3D

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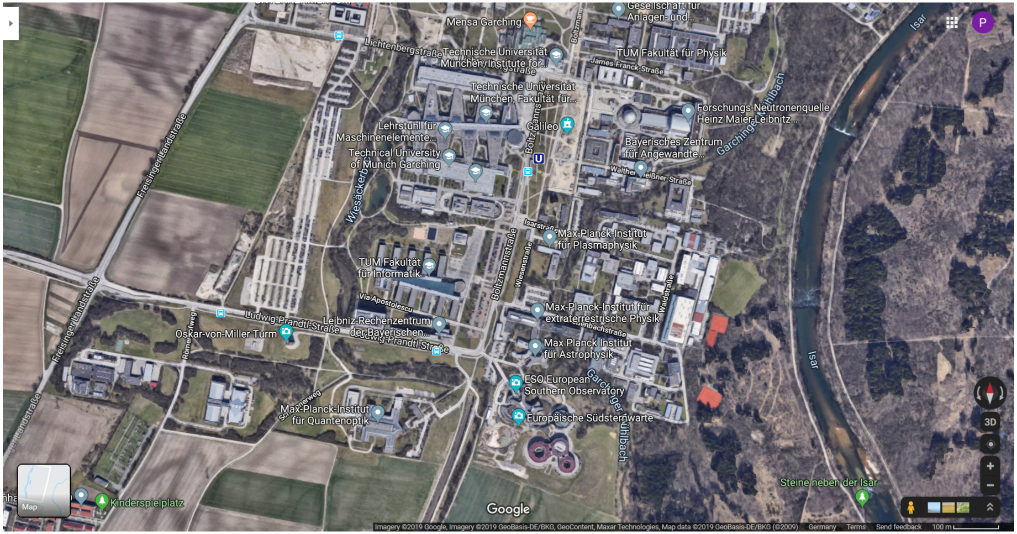
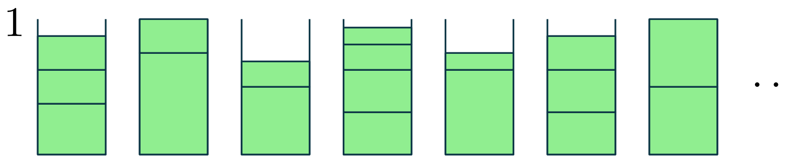


Connecting

Big Data Algorithms & **Combinatorial Optimization**



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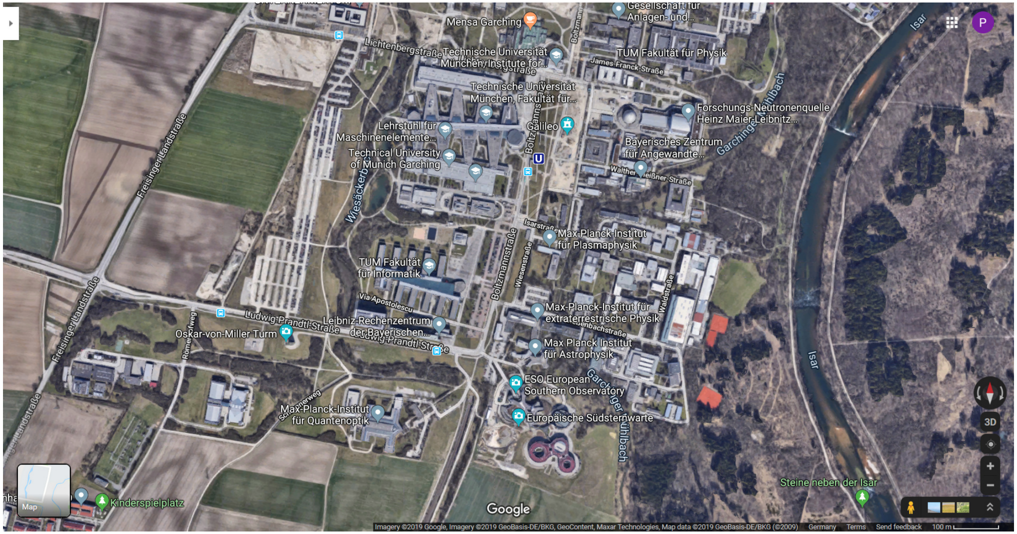

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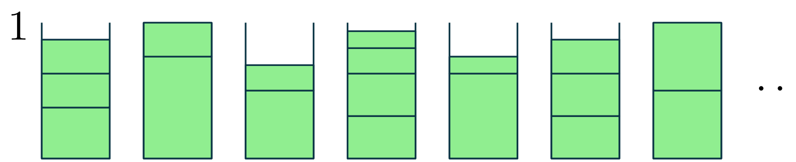

This talk's focus:

streaming algorithms | packing and scheduling

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- One pass over data w/ limited memory

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$\neq$ online

no need to make online decisions about the solution

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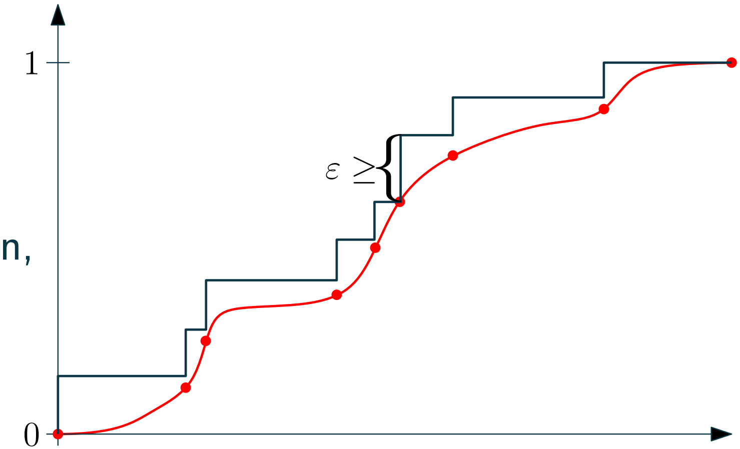
$$\mathrm{cdf}_A(x) = \frac{\{a \in A \,|\, a \leq x\}}{N}$$



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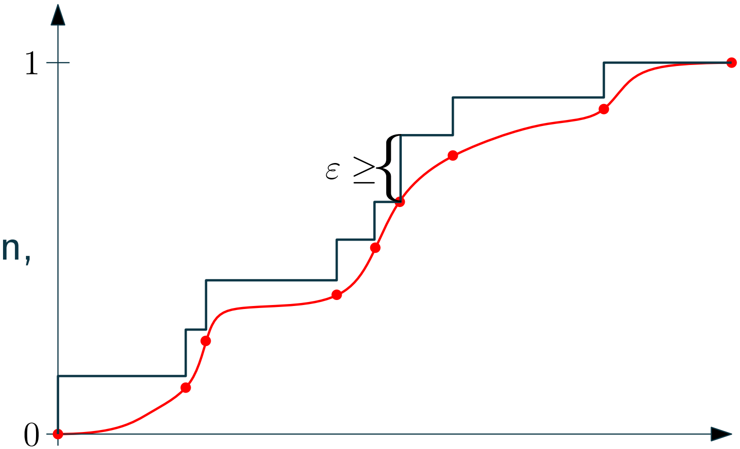
- some graph problems,
- submodular maximization,
- ...



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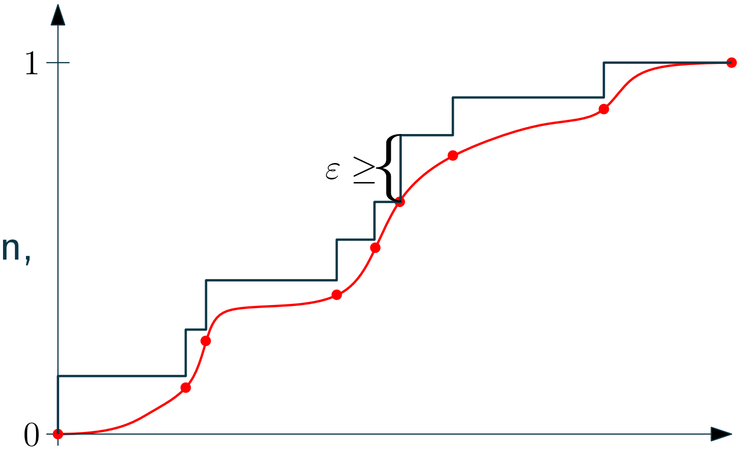
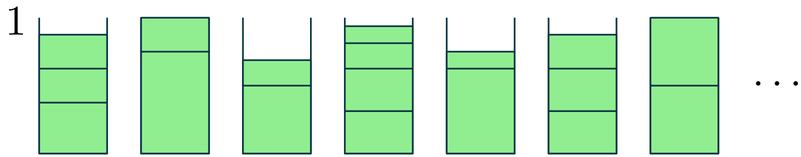


What about other basic problems in combinatorial optimization?

Our Results: Streaming Algorithms for ...

Bin Packing:



- Input: items of size in $[0, 1]$
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- Offline: $\mathrm{OPT} + \mathcal{O}(\log \mathrm{OPT})$ bins in poly-time [Hoberg, Rothvoss '17]

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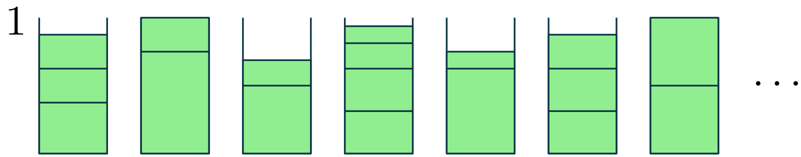

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Essentially best possible

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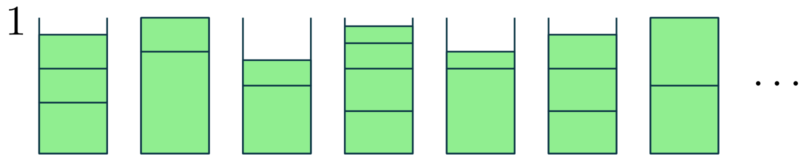

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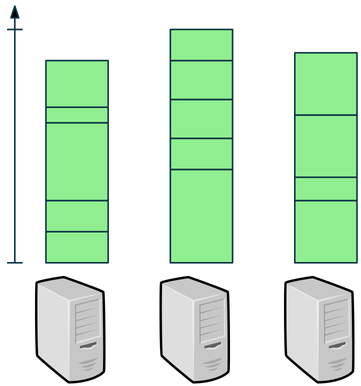

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$= $ maximum load over all machines

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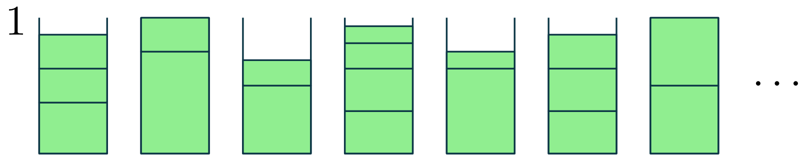

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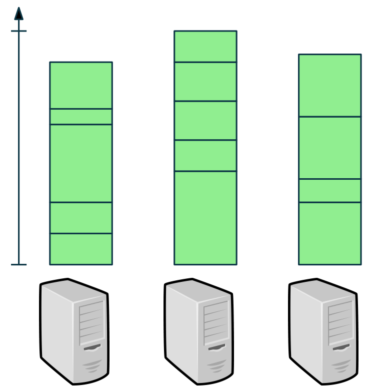

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- $1+\varepsilon$-approximation (rounding & DP)

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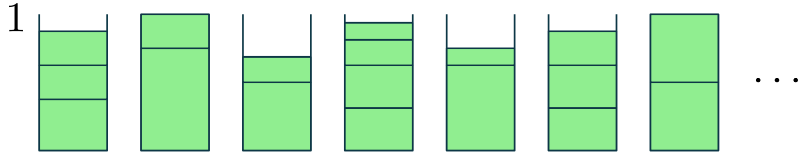

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Vector Scheduling:

- Input: jobs characterized by d-dim. vectors
  - e.g.: processing time, memory or bandwidth requirements, etc.
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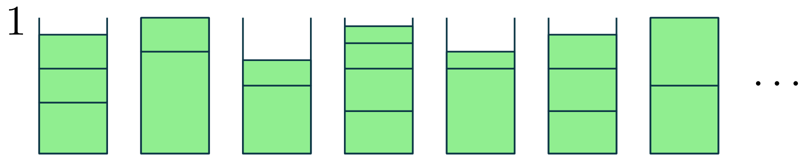

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2-approximation in space $\widetilde{\mathcal{O}}(d^2 \cdot m^3)$

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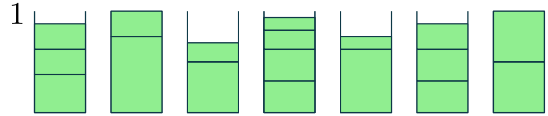

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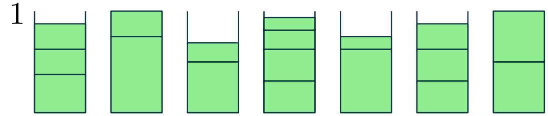

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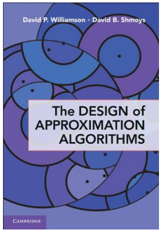

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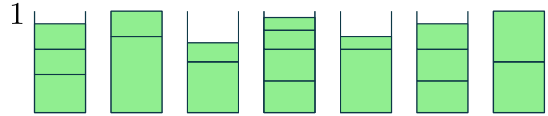

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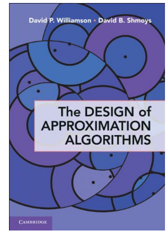

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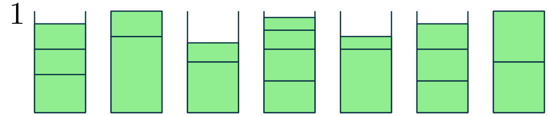

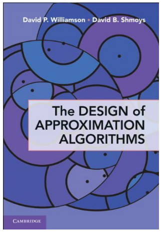

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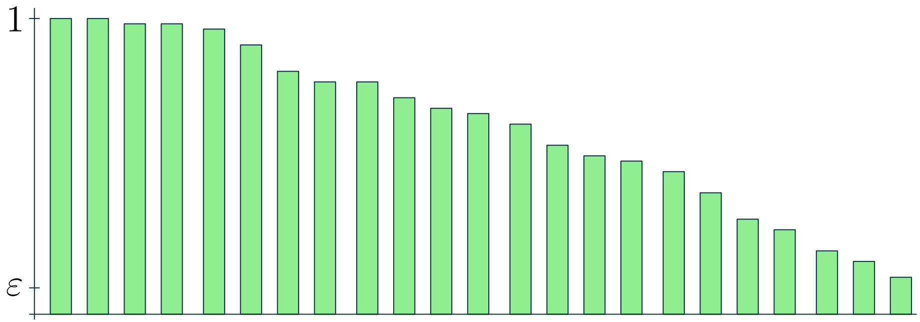

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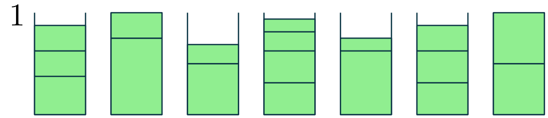

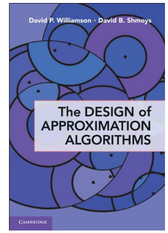

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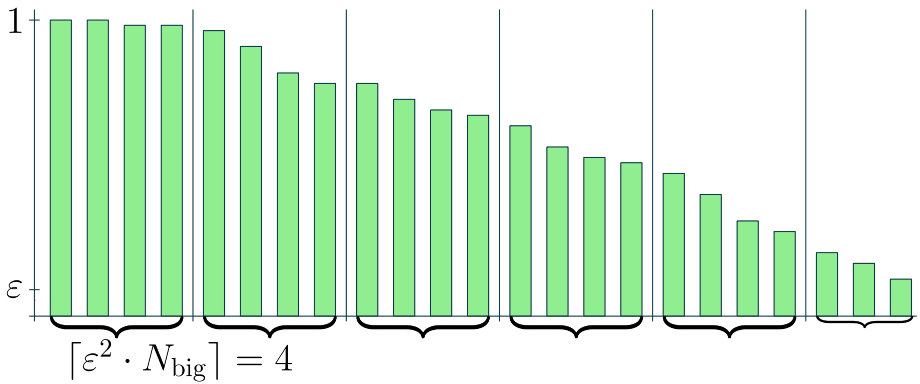

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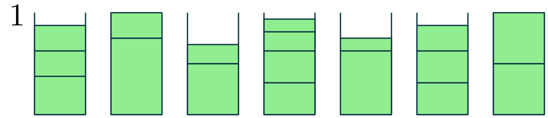

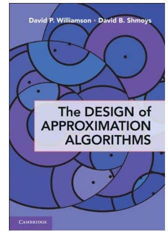

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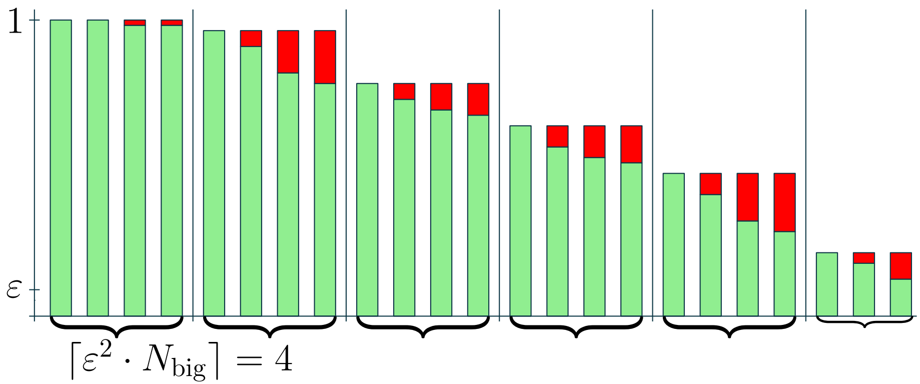

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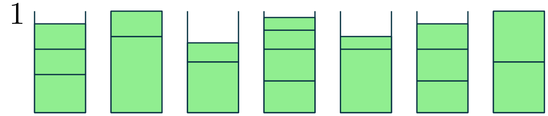

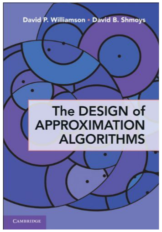

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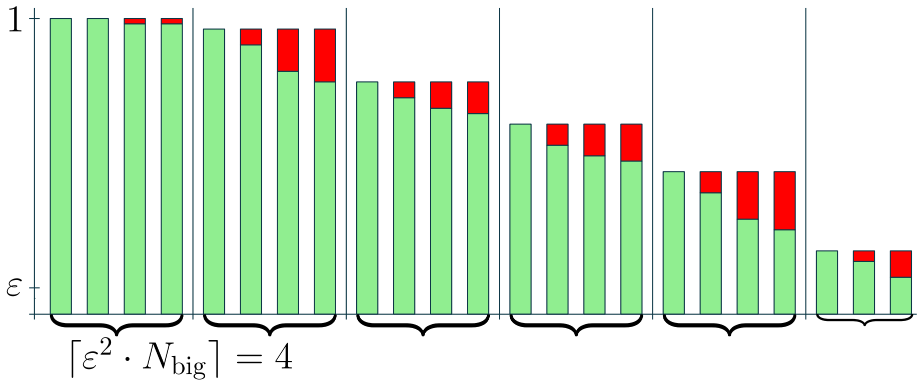

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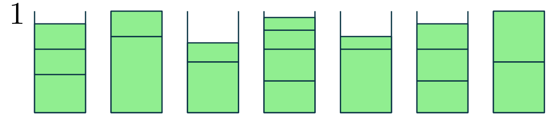

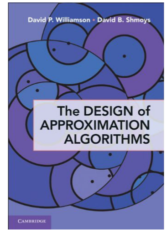

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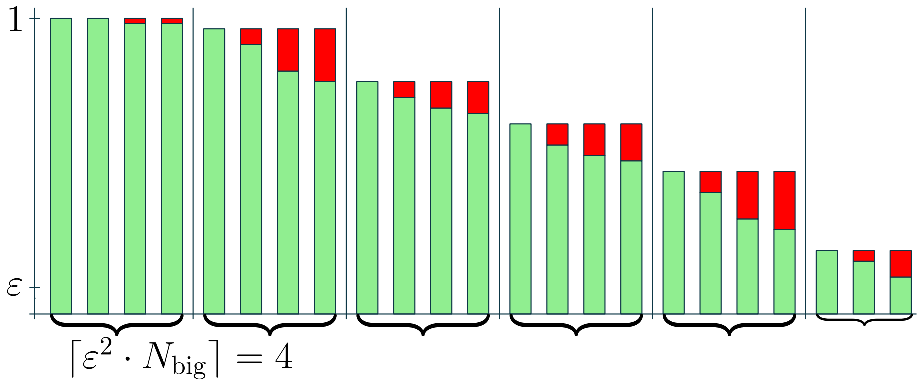

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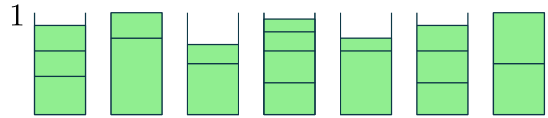

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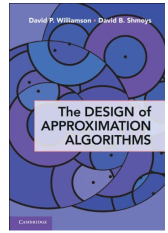

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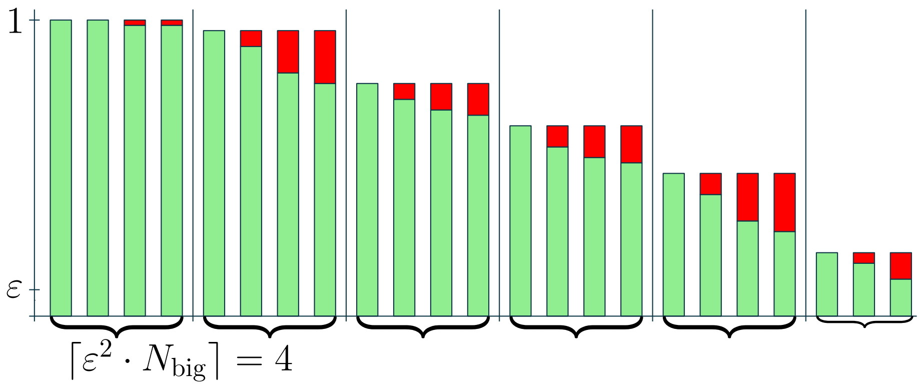

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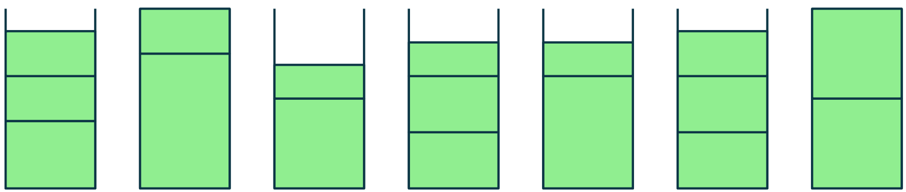

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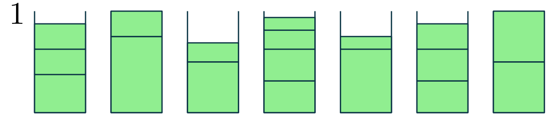
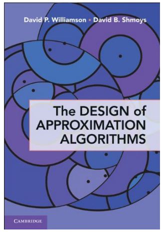
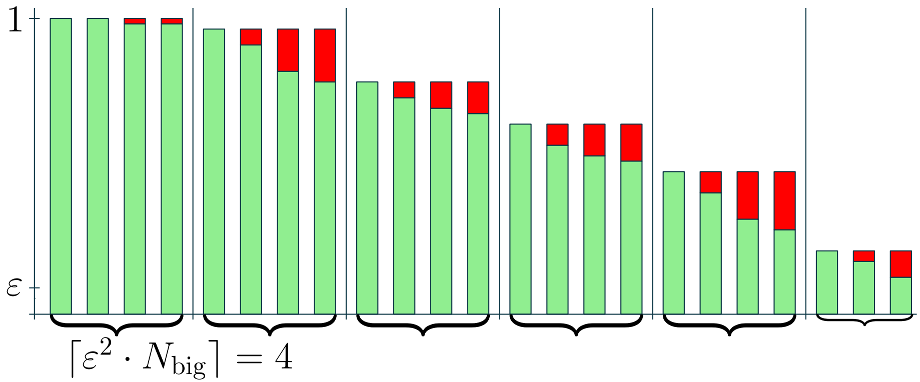


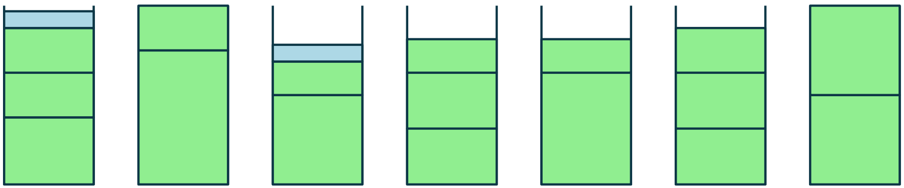


Textbook $1+\varepsilon$-approximation from [Fernandez de la Vega, Lueker '81]

- = solution with at most $(1+\varepsilon)\cdot \mathrm{OPT} + \mathcal{O}_\varepsilon(1)$ bins
- Big items: size $> \varepsilon$: linear grouping

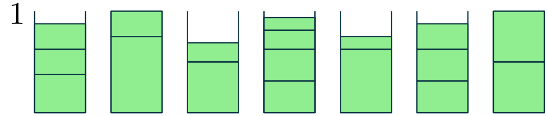


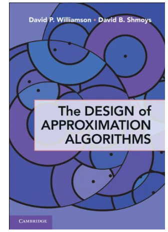
  - $\rightarrow$ instance with $\lceil \frac{1}{\varepsilon^2} \rceil$ item sizes only $\rightarrow$ solve (nearly) optimally
- Small items: size $\le \varepsilon$:
  - Fill in greedily



Bin Packing: Offline Approximation Scheme

- Input: items of size in $[0, 1]$
- Goal: pack into min. number of bins of capacity 1





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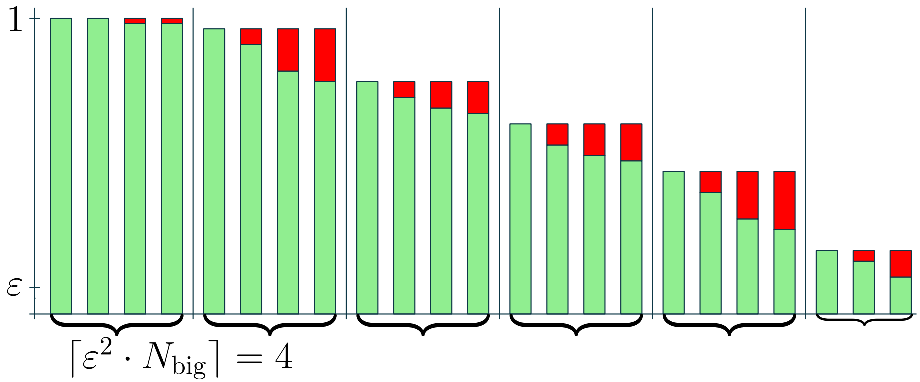

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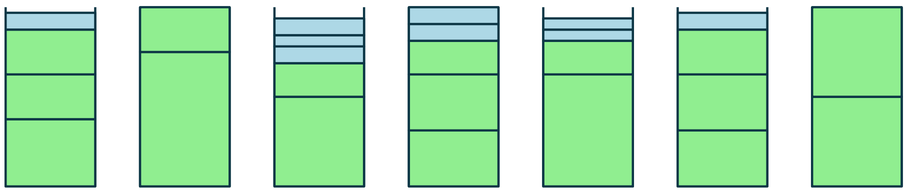

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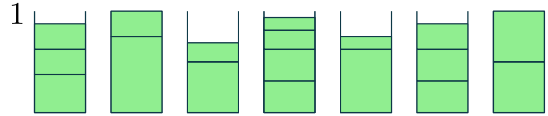

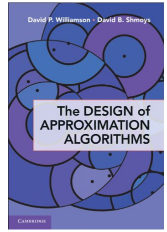

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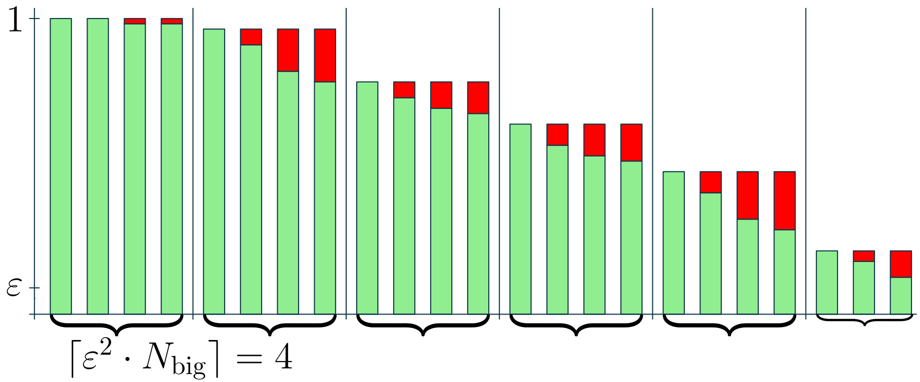

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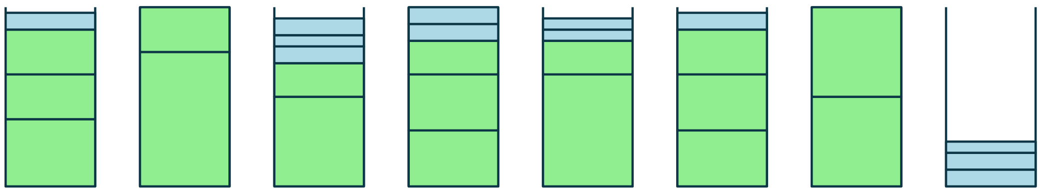

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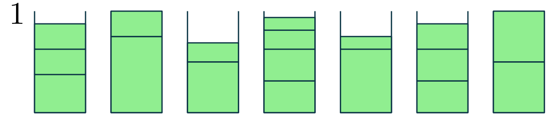

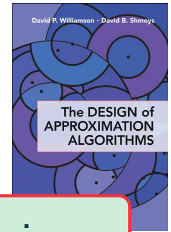

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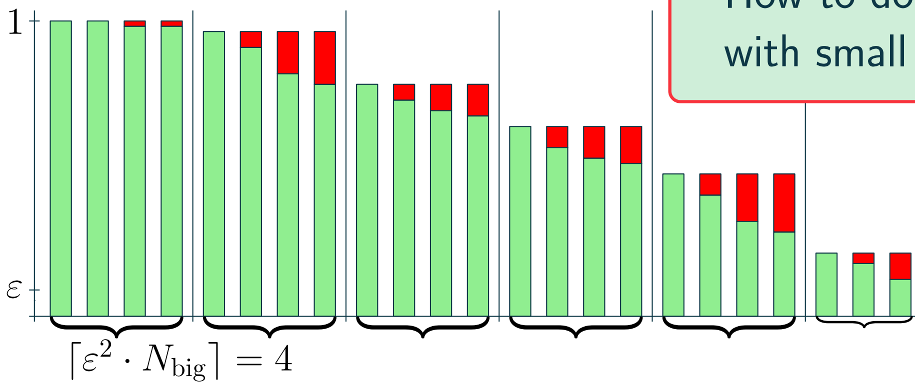
How to do grouping with small memory?



  - $\rightarrow$ instance with $\left\lceil \frac{1}{\varepsilon^2} \right\rceil$ item sizes only $\rightarrow$ solve (nearly) optimally
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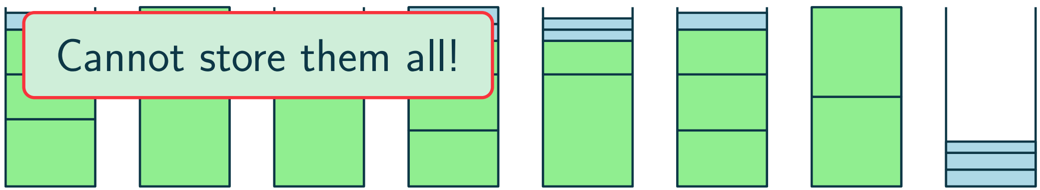
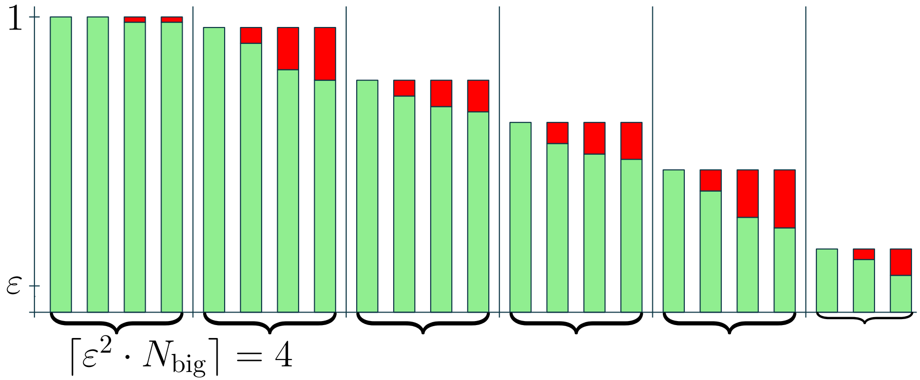
Cannot store them all!



Streaming $1 + \varepsilon$-Approx. for Bin Packing

Streaming $1+\varepsilon$-Approx. for Bin Packing

Big items: size $> \varepsilon$: **approximate linear grouping**



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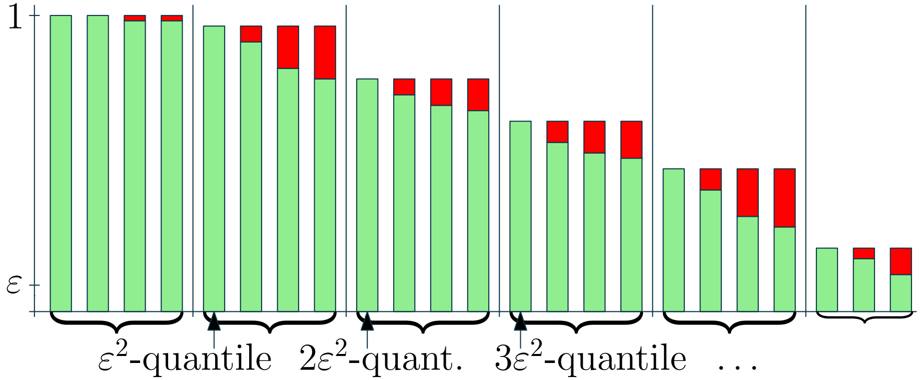

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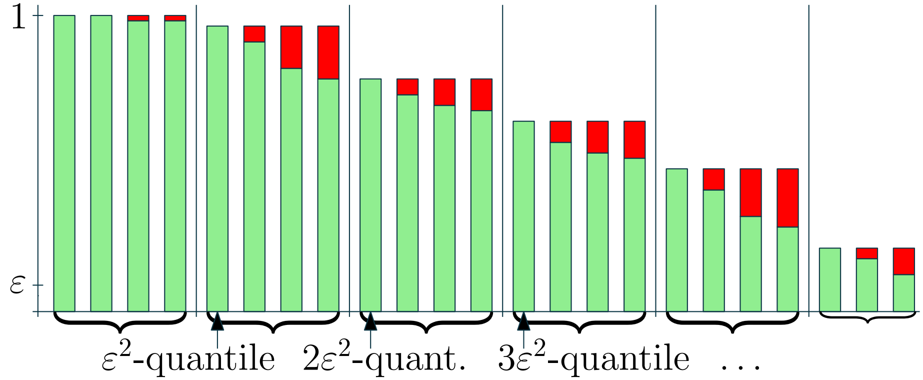

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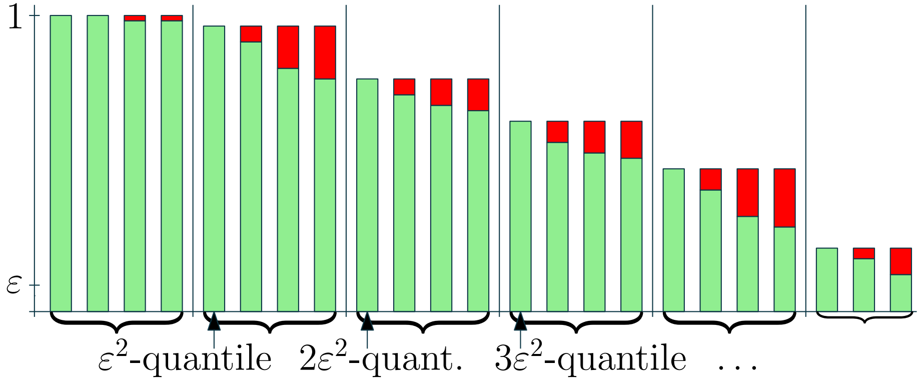

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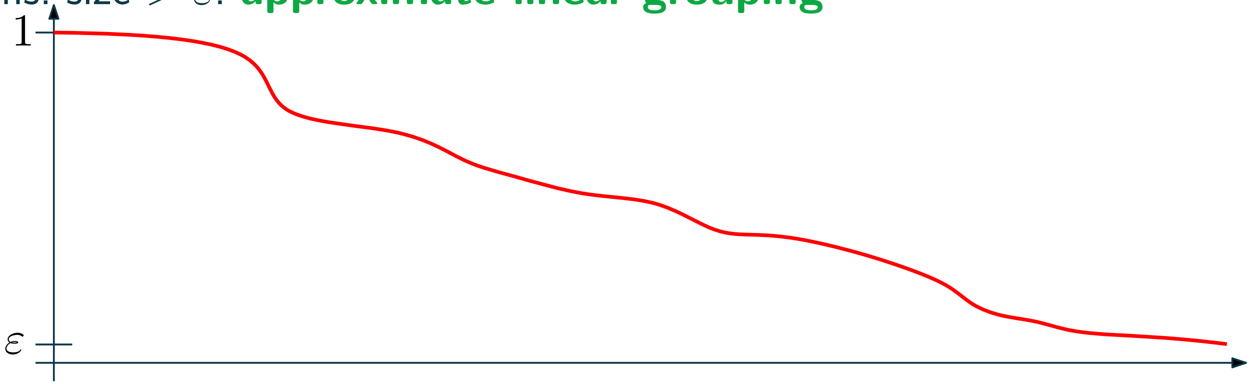

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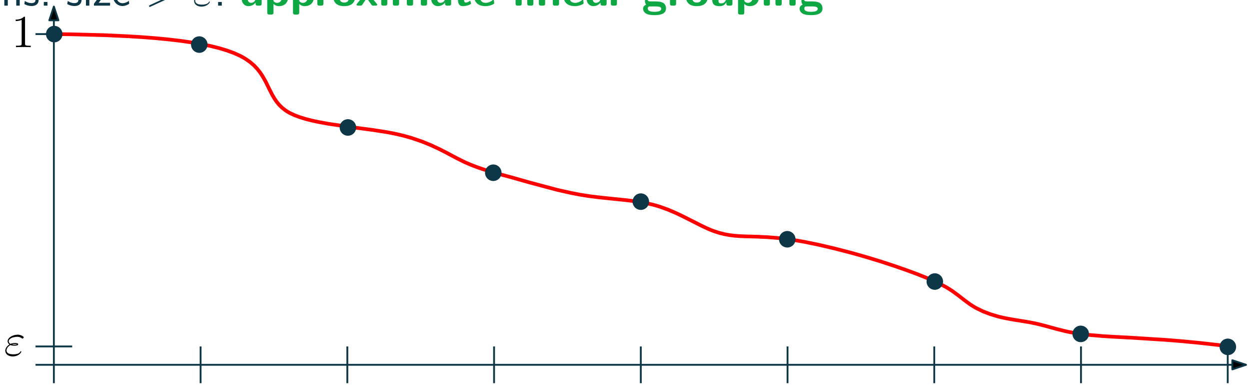

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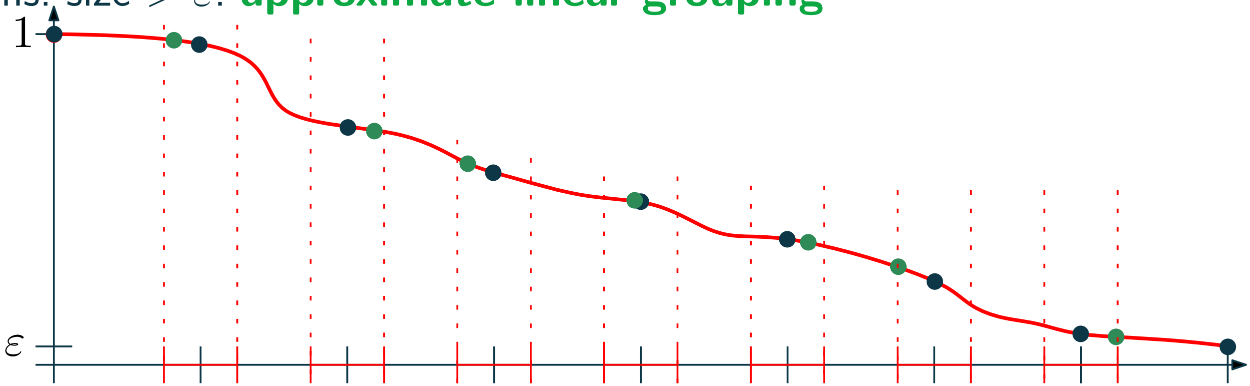

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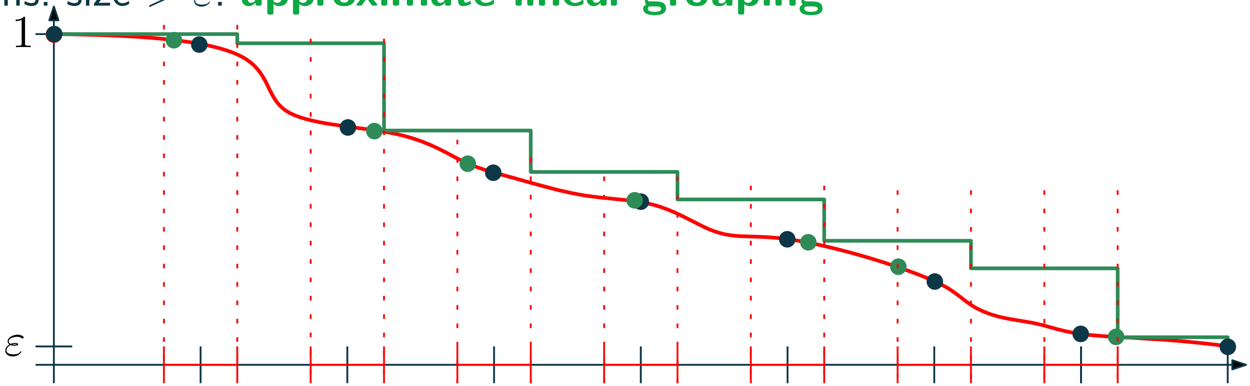

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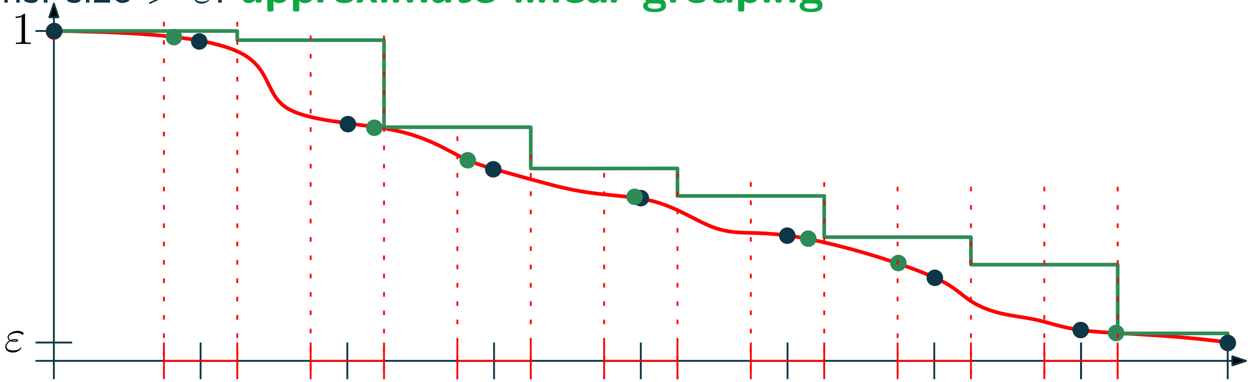

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Small items

- sum their sizes
- fill in fractionally

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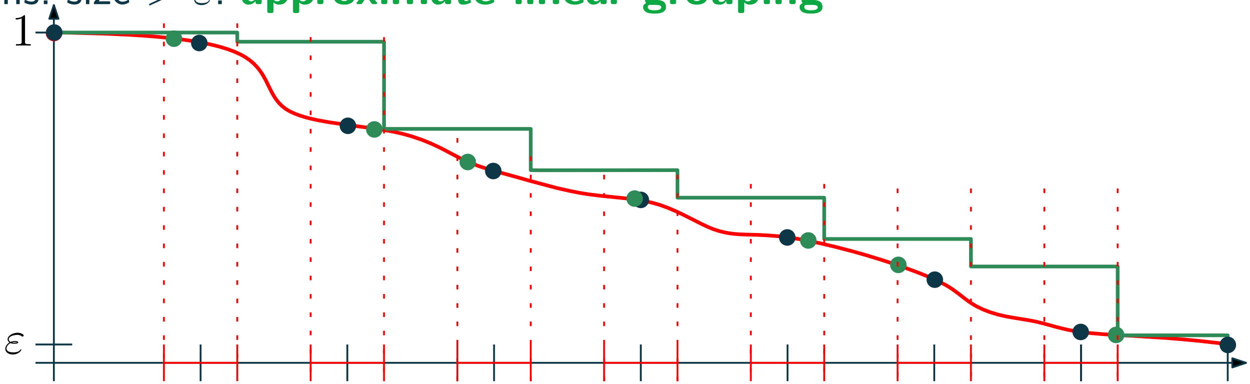

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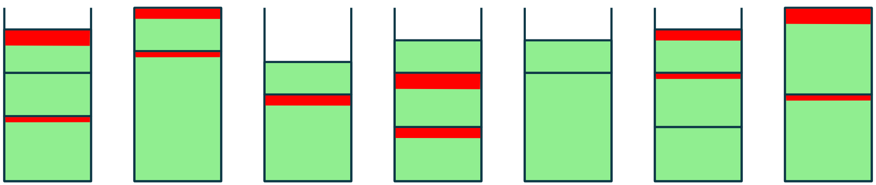

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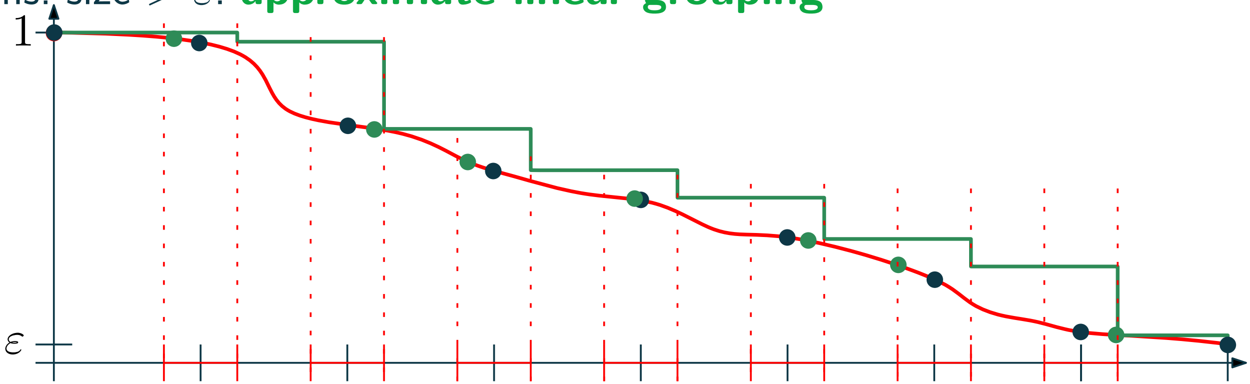

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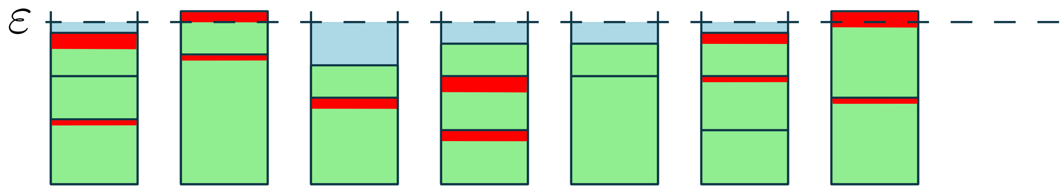

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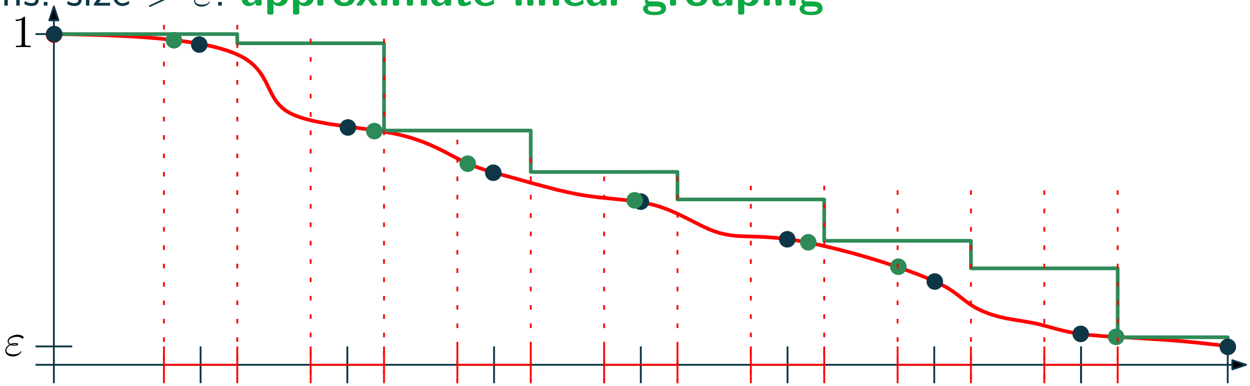

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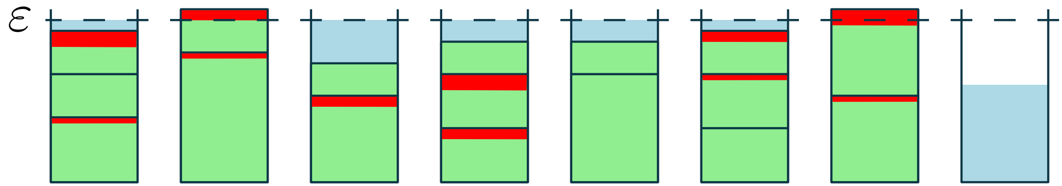

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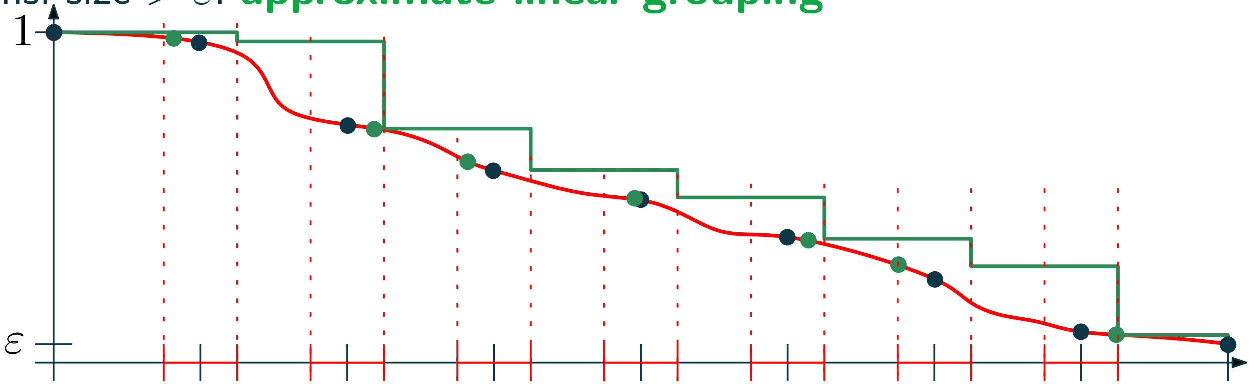

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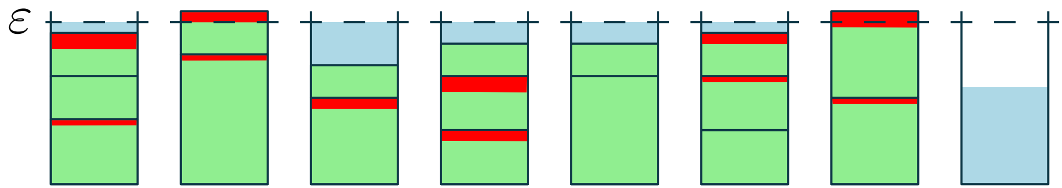

$\Rightarrow 1+\varepsilon$-approx. for Bin Packing in space $\mathcal{O}(\frac{1}{\varepsilon^2} \cdot \log \varepsilon \mathrm{OPT})$

Streaming $1 + \varepsilon$-Approx. for Bin Packing

Streaming $1+\varepsilon$-Approx. for Bin Packing

$1+\varepsilon$-approx. for $\textsc{Bin Packing}$ in space $\mathcal{O}(\frac{1}{\varepsilon^2} \cdot \log \varepsilon \mathsf{OPT})$

- by quantiles with precision $\approx \varepsilon^2$

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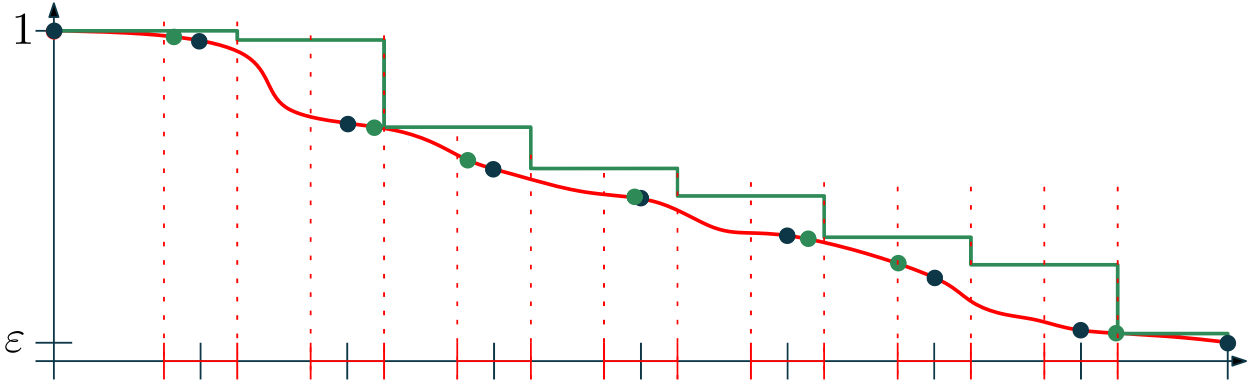

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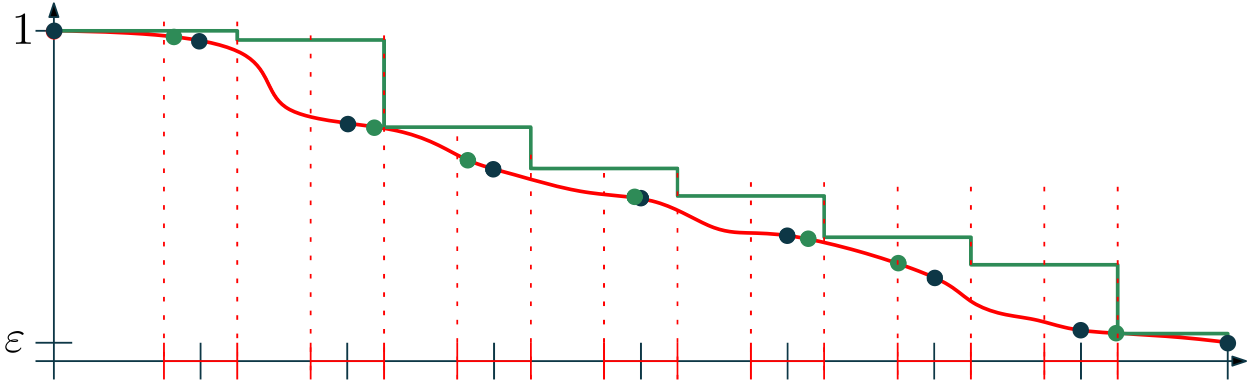

- too much precision for small items

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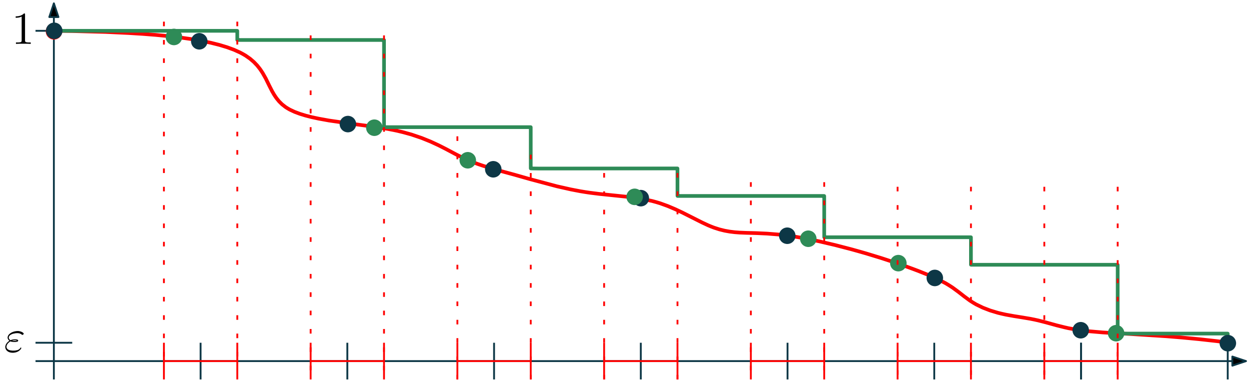

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Geometric grouping [Karmarkar, Karp '82]

- Split big items into $\lceil \log_2 \frac{1}{\varepsilon} \rceil$ size groups: $(\frac{1}{2}, 1], (\frac{1}{4}, \frac{1}{2}], \ldots$
- Use quantile summary for each group with precision $\approx \varepsilon$
- $\Rightarrow$ space $\mathcal{O}(\frac{1}{\varepsilon} \cdot \log \frac{1}{\varepsilon} \cdot \log \mathrm{OPT})$

Streaming $1+\varepsilon$-Approx. for Bin Packing

Can we do better than $\mathcal{O}(\frac{1}{\varepsilon} \cdot \log \frac{1}{\varepsilon} \cdot \log \mathrm{OPT})$?

Streaming $1+\varepsilon$-Approx. for Bin Packing

Can we do better than $\mathcal{O}(\frac{1}{\varepsilon} \cdot \log \frac{1}{\varepsilon} \cdot \log \mathrm{OPT})$?

Yes! If . . .

- . . . items drawn from a bounded-size universe U
  - space $\mathcal{O}(\frac{1}{\varepsilon} \cdot \log \frac{1}{\varepsilon} \cdot \log |U|)$ using a quantile summary from [Shrivastava *et al.* '04]

Streaming $1+\varepsilon$-Approx. for Bin Packing

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- . . . randomization allowed (wrong answer w/ probability γ)
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No, not much by a deterministic comparison-based algo.

- Streaming $1 + \varepsilon$-approx. for Bin Packing in space S

  $\Rightarrow$ estimating rank w/ accuracy $\approx \varepsilon$ in space S

Streaming $1+\varepsilon$-Approx. for Bin Packing

Can we do better than $\mathcal{O}(\frac{1}{\varepsilon} \cdot \log \frac{1}{\varepsilon} \cdot \log \mathrm{OPT})$?

Yes! If ...

- ... items drawn from a bounded-size universe U
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- Streaming $1+\varepsilon$-approx. for Bin Packing in space S

  $\Rightarrow$ estimating rank w/ accuracy $\approx \varepsilon$ in space S
- LB $\Omega(\frac{1}{\varepsilon} \cdot \log \varepsilon N)$ for estimating rank / quantile summaries [Cormode & **V.** '19+]

  $\Rightarrow$ LB $\Omega(\frac{1}{\varepsilon} \cdot \log \mathrm{OPT})$ for Bin Packing

Vector Scheduling: Rounding

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- Input: jobs characterized by *d*-dimensional vectors
- Goal: assign jobs to machines to minimize makespan

  = maximum load over all machines and dimensions
- $1 + \varepsilon$-approximation (more involved rounding & MIP) [Bansal *et al.* '16]

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Makespan Scheduling: $d = 1$

- Rounding currently big jobs to powers of $1 + \varepsilon$
  - Big jobs $=$ bigger than ε times currently largest job

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Vector Scheduling: $d > 1$

- More intricate rounding from [Bansal *et al.* '16]:
  - Round to 0 coordinates small relatively to $\|v\|_\infty$
  - Big jobs: round each dimension to power of $1 + \varepsilon \Rightarrow$ space $\widetilde{\mathcal{O}}(\frac{1}{\varepsilon})^d$
  - Small jobs: round relative to $\|v\|_\infty$

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Job vector v big if $\|v\|_\infty > \gamma\cdot$ (LB on OPT) for $\gamma \approx \varepsilon^2 / \log \frac{d}{\varepsilon}$

- $\Rightarrow$ at most $d \cdot m/\gamma$ big vectors
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tight!

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$O(1)$-approx. in space poly(d)?

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Thank You!

Streaming vs. Online

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Problem	Streaming apx.	Competitive ratio LB	Competitive ratio UB
Bin Packing	$1+\varepsilon$	1.542 [Balogh *et al.* '19]	1.578 [Balogh *et al.* '18]
Vector Bin Packing	$d+\varepsilon$	$\Omega(d^{1-\varepsilon})$ [Azar *et al.* '13]	$d+0.7$ [Garey *et al.* '76]
Makespan Scheduling	$1+\varepsilon$	1.88 [Rudin '01]	1.92 [Fleischer & Wahl '00]
Vector Scheduling	$2\,(1+\varepsilon)$	$\Omega(\log d/\log\log d)$ [Im *et al.* '15]	$\mathcal{O}(\log d/\log\log d)$ [Im *et al.* '15]