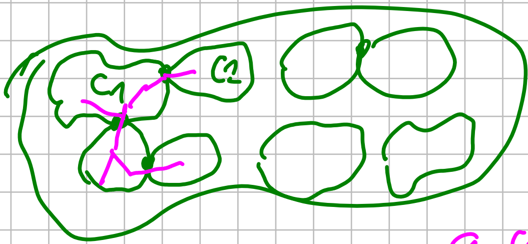
- Fredman–Tarjan (iterated Jarník): $O(m)$ if $\rho \geq \log^{(k)} n$
- Borůvka steps: each step runs in $O(m)$, $n' \leq \frac{n}{2}$, $m' \leq m$
- Topological Graph Computations
- **Partioning**

G, t, ε



$|R^C| \leq 2\varepsilon m$

R^C = set of corrupted edges

$\forall i\ |V(C_i)| \leq t$

a component of $\bigcup C_i$ has $\geq t$ vertices

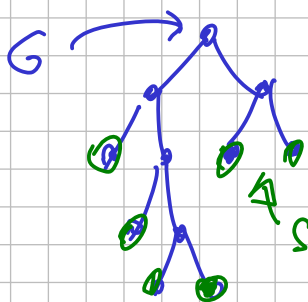
$C_1 \;—\; C_k$

$$msf(G) \subseteq \underbrace{\bigcup_i msf(C_i) \cup msf\big(\underbrace{G / \textstyle\bigcup_i G_i \setminus R^C}_{G_A}\big)}_{G_B} \cup R^C$$

$G/\bigcup_i C_i$ at least t times smaller

$O(m)$ for ε constant

- **Decision Trees**



correct
depth ≥ time
optimal

$\mathcal{D}(G)$ opt. D.T. for G

$D(G)$ depth of $\mathcal{D}(G)$

$D(m,n) = \max D(G)$ over all such G

← lower bound on time complexity of all comparison-based algs

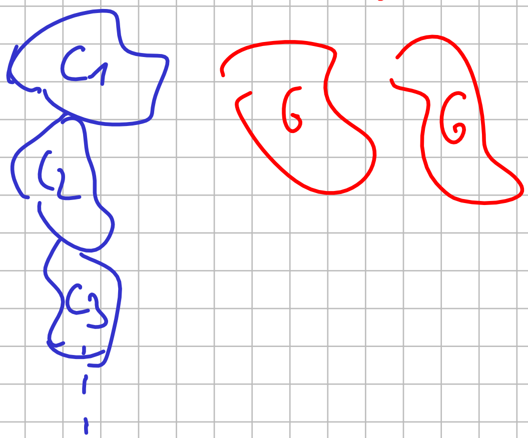
Goal: Achieve time $O(D(m,n))$

$D(m,n) \leq 4/3 \cdot n^2$

Brute-force $\mathcal{D}(G)$ in $O\big(\underbrace{2^{2^{4n^2}}}\big)$ time.

1. $D(m,n) \geq m/2$
2. $D(m',n') \geq D(m,n)$ for $m' \geq m$, $n' \geq n$
3. $D(G) = \sum_i D(C_i)$
4. $\underbrace{2D(m,n)}_{D(G)+D(G)} \leq D(2m,2n)$

[Compartments
$msf(G) = \bigcup_i msf(C_i)$



$t \leftarrow \lfloor \log\log\log n \rfloor$

Partition with $\varepsilon = 1/8$ $\longrightarrow$ $C_1 — C_k$ $\quad |C_i| \le t$; R^C $\quad |R^C| \le \frac{1}{4} m$

Build optimum D.T.

$\forall i$ $F_i \leftarrow msf(C_i)$ using D.T. $\leftarrow$ Topological comp.

$G_A \leftarrow \left(G / \bigcup_i C_i\right) \setminus R^C$

$F_A \leftarrow msf(G_A)$ using Fredman–Tarjan

$G_B \leftarrow \bigcup_i F_i \cup F_A \cup R^C$ $\quad |G_B| \le n + \frac{m}{4}$ $\quad$ set of contracted edges

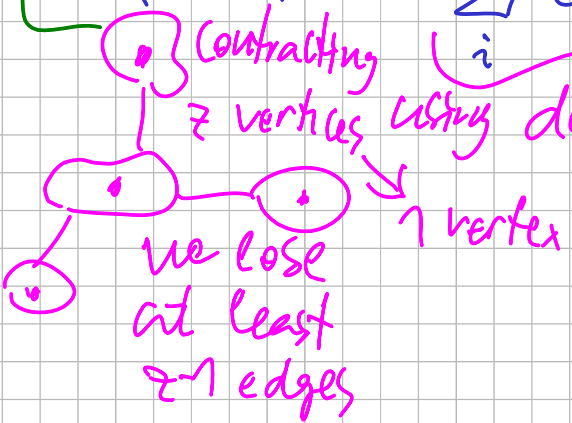
Run 2 Borůvka steps on G_B $\rightarrow$ G_C, F_B

Return $F_B \cup \underbrace{mst(G_C)}_{F_C}$ $\quad$ obtained recursively

Lemma

$$\boxed{T(m,n) \le \underbrace{\sum_i c_1 D(C_i)}_{\text{using decision trees}} + \underbrace{T(m/2, n/4)}_{\text{final recursion}} + \underbrace{c_2 m}_{\text{everything else}}}$$

$mst(G)$



$n \rightarrow n/4$

we lost $\ge \frac{3}{4} n$ vertices

we lost $\ge \frac{3}{4} n$ edges

$n + \frac{m}{4} \rightarrow \le \underbrace{\frac{n}{4}}_{\le \frac{m}{4}} + \frac{m}{4} \le \frac{1}{2} m$

Guess: $\underbrace{T(m,n) \le c \cdot D(m,n)}$

$$\boxed{\text{All on DM ?}}$$

MST

$D(m,n) \le m \cdot \alpha(m,n)$

Pettie $\quad$ Chazelle's alg.