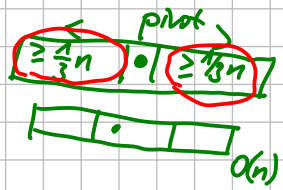
Soft Heap

$\varepsilon \in (0, 1/2]$ corruption

current ≥ original key

At any moment: # corrupted ≤ $\varepsilon \cdot n$

n = # items inserted so far



Operations:
- Create(ε, x)
- Insert(x)
- ExtractMin
- Meld
- Explode / Dismantle

} $O(\log 1/\varepsilon)$ amortized time

} $O(1)$ am. time (per item)

$\varepsilon = \frac{1}{2}$

An Example

1. $\varepsilon \leftarrow 1/3$
2. Insert all items
3. ExtractMin $n/3$ times
4. Last extracted item → p

- $p \geq$ current values of extracted items $\geq$ orig. values of extracted items
- $p \leq$ current values of $\frac{2}{3}n$ remaining items (at most $\frac{1}{3}n$ corrupted) $\geq$ orig. values for $\geq \frac{n}{3}$ remaining items

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Versions: ① Chazelle ② M.M. ③ Kaplan&Zwick

Binary trees

rank(child) = rank(parent) − 1

tree heap-ordered by ckeys

list(x) := doubly linked list of items

ckey(x) ↑ controlling key

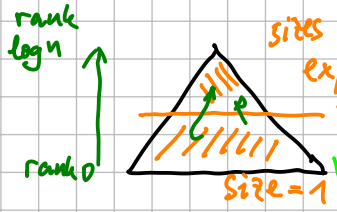
current key ≥ orig. key of all items in list(x)

size(x) ... expected |list(x)| ↑ depends on rank

s_k = size for x of rank k

$$s_k = \begin{cases} 1 & \text{if } k \leq r \\ \left\lceil \frac{3}{2} \cdot s_{k-1} \right\rceil & \text{if } k > r \end{cases}$$

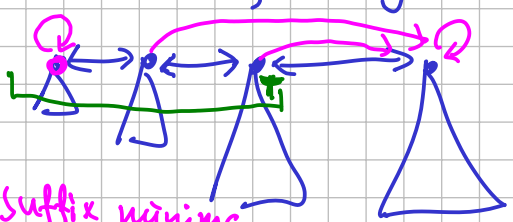
$r := \lceil \log 1/\varepsilon \rceil + 5$



By induction:

$$\left(\frac{3}{2}\right)^{k-r} \leq s_k \leq 2 \cdot \left(\frac{3}{2}\right)^{k-r} - 1 \quad \text{for } k > r$$

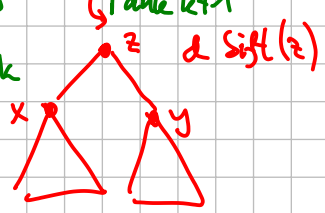
Heap: a sequence of trees of strictly increasing ranks



suffix minima

combine(x, y) → root of rank k+1

roots of rank k



& Sift(z)

USM(T) update suffix minima before T

Create: 1 node of rank 0

Meld:

Insert: Create & Meld

Sift(x) ... list(x) refill

x not a leaf

1. Ensure that ckey(left(x)) ≤ ckey(right(x)) (+∞ if no right child) [swap L/R if needed]
2. Move all from list(left(x)) to list(x)
3. Update ckey(x) to ckey(left(x))
4. Sift(left(x)) if left(x) is not a leaf (or remove left(x) if a leaf)
5. Repeat Sift(x) if |list(x)| < size(x)

ExtractMin

- uses suffix min of the 1st tree → root with min ckey
- extract item from list(x)
- if $|list(x)| < \frac{1}{4}$ size(x): Sift(x) & x is not leaf & USM(x)

concat of lists: $\leq s_k$ dest. $\leq 3 \cdot s_{k-1}$ src. $\leq \frac{2}{3} s_k$ $\leq 3 s_k$ $\leq 2 s_k$

Invariant: If $\text{rank}(x) \leq r$: $|list(x)| = 1$

If $\text{rank}(x) > r$ & x is not a leaf: $\frac{1}{2} size(x) \leq |list(x)| \leq 3 \cdot size(x)$

Inv.: # nodes of rank k $\leq n / 2^k$

Inv: # corrupted items $\leq \varepsilon n$

Amortized Analysis: TODO