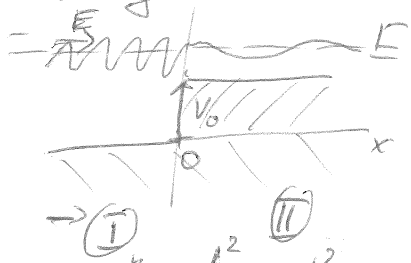
10 bariéry

X

ÚKMTS.1

| schod TS.1 & TS1b
| koeficient TS.2 & TS.4
| schod $E>V$: TS.2b, TS.3, TS3b
| ~~koeficien~~
| schod $E<V$ - součástka - TS.4b, TS.5
- vznícení TS.6 TS.7
δ potenciál - TS.8



$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V\psi = E\psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + (V-E)\psi = 0$$

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + (E-V)\psi = 0$$

E "libovolné"
- "okrajová podmínka"

oblast I $\quad V=0$

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + E\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

uhodneme řešení $\quad e^{\pm ikx} \qquad k = \frac{\sqrt{2mE}}{\hbar}$

oblast II $\quad V=V_0$

elektrony $\rightarrow$ $\quad e^{\pm ik'x} \qquad k' = \frac{\sqrt{2m(E-V_0)}}{\hbar}$

V_0

odraz

$$\psi = N(e^{ikx} + r e^{-ikx}) \qquad x \le 0$$

$$\psi = N s e^{ik'x} \qquad x \ge 0$$

spojitost $\quad N(e^{ikx} + r e^{-ikx})\big|_{x=0} = N s e^{ik'x}\big|_{x=0}$

$$1 + r = s$$

spojitost derivace

$$N(ik e^{ikx} - r ik e^{-ikx})\big|_{x=0} = N s\, ik' e^{ik'x}\big|_{x=0}$$

$$k - rk = sk'$$

$$k(1-r) = sk'$$

$1+r=s$ $\quad$ X $\quad$ UKM TS.13

$k(1-r)=sk'$

$k=\frac{\sqrt{2mE}}{\hbar}$; $k'=\frac{\sqrt{2m(E-V_0)}}{\hbar}$

$V_0=cE \quad 0<c<1$

$$k'=\frac{\sqrt{2m(E-cE)}}{\hbar}=\frac{\sqrt{2mE}}{\hbar}\sqrt{1-c}=k\sqrt{1-c}$$

$1+r=s$

$k(1-r)=sk\sqrt{1-c}$

$(1-s+1)=s\sqrt{1-c}$

$2=s\sqrt{1-c}+s$

$2=s(1+\sqrt{1-c})$

$s=\frac{2}{1+\sqrt{1-c}}$

$1+r=\frac{2}{1+\sqrt{1-c}}$

$r=\frac{2}{1+\sqrt{1-c}}-1=\frac{2-1-\sqrt{1-c}}{1+\sqrt{1-c}}=$

$=\frac{1-\sqrt{1-c}}{1+\sqrt{1-c}}$

$T=\frac{k'}{k}s^2$

$R=r^2=\frac{(1-\sqrt{1-c})^2}{(1+\sqrt{1-c})^2}=\frac{1-2\sqrt{1-c}+1-c}{1+2\sqrt{1-c}+1-c}=$

$T=\sqrt{1-c}\,\frac{4}{1+2\sqrt{1-c}+1-c}$

$=\frac{2-c-2\sqrt{1-c}}{2-c+2\sqrt{1-c}}$

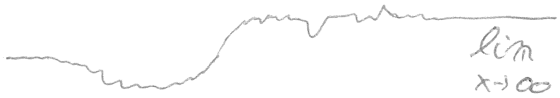
$=\frac{4\sqrt{1-c}}{2-c+2\sqrt{1-c}}$

$c=\frac{3}{4}$ $\quad \sqrt{1-c}=\sqrt{1-\frac{3}{4}}=\frac{1}{2}$

$T=\frac{2-3/4-2\cdot 1/2}{2-3/4+2\cdot 1/2}=\frac{1-3/4}{3-3/4}=\frac{1/4}{9/4}=\frac{1}{9}$ $\quad$ OK

$R=\frac{8}{9}$

21. 11. 1 ✗ ÚKM T5.2



$\lim\limits_{x\to\infty} V(x) = V_0$

$\lim x \to -\infty : V(x) = 0$

$\psi = e^{ikx} + Be^{-ikx}$ $\quad x \to -\infty$ $\qquad \psi = A e^{ik'x} \quad x \to \infty$

$k = \sqrt{\frac{2mE}{\hbar^2}} \qquad k' = \frac{\sqrt{2m(E-V_0)}}{\hbar}$

$$j = \frac{\hbar}{2mi}\left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx}\right)$$

pro $x \to \infty$ $\quad j_\infty = \frac{\hbar}{2mi}\left(A^* e^{-ik'x} A e^{ik'x}(ik') - AA^* e^{ik'x}(-ik')e^{-ik'x}\right)$

$$j_\infty = \frac{\hbar |A|^2}{2m} 2k' = \frac{\hbar k' |A|^2}{m}$$

pro $x \to -\infty$ $\quad j_{-\infty} = \frac{\hbar}{2mi}\Big[\left(e^{-ikx} + Be^{ikx}\right)\left(ike^{ikx} - Bike^{-ikx}\right) -$

$$-\left(e^{ikx} + Be^{-ikx}\right)\left(-ike^{-ikx} + ikBe^{ikx}\right)\Big]$$

$$j_{-\infty} = \frac{\hbar}{2m}\Big[\left(k - \cancel{Bke^{-2ikx}} + \cancel{Bke^{2ikx}} - B^2k\right) -$$

$$-\left(-k + \cancel{Bke^{2ikx}} - \cancel{Bke^{-2ikx}} + B^2k\right)\Big]$$

$$j_{-\infty} = \frac{\hbar k}{\cancel{2}m}\left[1 - B^2\right]$$

T5.4 viz

„stacionární stav" $\frac{d|\psi|^2}{dt} = 0 \Rightarrow \frac{dj}{dt} = 0$ viz Lin 1054 $\quad \frac{dj}{dx} = 0 \to j = \text{const}$

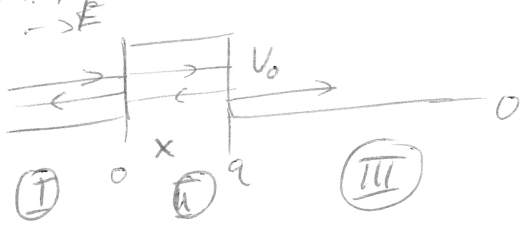
$j(-\infty) = j(\infty) \leftarrow$ $\qquad \frac{\partial \rho}{\partial t} + \operatorname{div} j = 0$

co přiteče, musí projít $\qquad \underset{=0}{\uparrow} \qquad \operatorname{div} j = 0$

$$\frac{\cancel{\hbar} k' |A|^2}{\cancel{m}} = \frac{\cancel{\hbar} k}{\cancel{m}}\left[1 - |B|^2\right]$$

$$\underbrace{\frac{k'}{k}|A|^2}_{\text{Transmise projde}} + \underbrace{|B|^2}_{\text{odráží = Reflexe}} = 1$$

21.11.4 $\quad$ X $\quad$ OKMTS.26



I: $\psi_I = c_1 e^{ikx} + c_2 e^{-ikx}$ $\qquad k = \frac{\sqrt{2mE}}{\hbar}$

$E > V_0$: II: $\psi_{II} = c_3 e^{ik'x} + c_4 e^{-ik'x}$ $\qquad k' = \frac{\sqrt{2m(E-V_0)}}{\hbar}$

III: $\psi_{III} = c_5 e^{ikx}$ $\quad \leftarrow$ jen prošla

$\psi_I(0) = \psi_{II}(0)$: $c_1 + c_2 = c_3 + c_4$

$\psi_I'(0) = \psi_{II}'(0)$: $ik(c_1 - c_2) = ik'(c_3 - c_4)$

$\psi_{II}(a) = \psi_{III}(a)$: $c_3 e^{ik'a} + c_4 e^{-ik'a} = c_5 e^{ika}$

$\psi_{II}'(a) = \psi_{III}'(a)$: $ik'c_3 e^{ik'a} - ik'c_4 e^{-ik'a} = ikc_5 e^{ika}$

jeden koeficient volíme – v důsledku normalizace $c_5 = 1$

$$c_1 + c_2 - c_3 - c_4 = 0$$
$$c_1 - c_2 - \frac{k'}{k}c_3 + \frac{k'}{k}c_4 = 0$$
$$e^{ik'a}c_3 + e^{-ik'a}c_4 = e^{ika}$$
$$k'e^{ik'a}c_3 - k'e^{-ik'a}c_4 = ke^{ika} \quad \leftarrow e^{ik'a}c_3 - e^{-ik'a}c_4 = \frac{k}{k'}e^{ika}$$

$$2e^{ik'a}c_3 = e^{ika}\left(1 + \frac{k}{k'}\right) \qquad c_3 = \frac{1}{2}e^{i(k-k')a}\left(1 + \frac{k}{k'}\right)$$
$$2e^{-ik'a}c_4 = e^{ika}\left(1 - \frac{k}{k'}\right) \qquad c_4 = \frac{1}{2}e^{i(k+k')a}\left(1 - \frac{k}{k'}\right)$$

$$c_1 + c_2 - \frac{1}{2}e^{i(k-k')a}\left(1 + \frac{k}{k'}\right) - \frac{1}{2}e^{i(k+k')a}\left(1 - \frac{k}{k'}\right) = 0$$
$$c_1 - c_2 - \frac{1}{2}\frac{k'}{k}e^{i(k-k')a}\left(1 + \frac{k}{k'}\right) + \frac{1}{2}\frac{k'}{k}e^{i(k+k')a}\left(1 - \frac{k}{k'}\right) = 0$$

$$\cancel{c_1 + c_2 - e^{i(k-k')a} = 0}$$
$$\cancel{c_1 - c_2 - \frac{k'}{k}e^{i(k-k')a}\frac{k}{k'} = 0}$$

21.11.4 contn'd

X

UKMTS.3

$$c_3=\frac{1}{2}e^{i(k-k')a}\left(1+\frac{k}{k'}\right)$$

$$c_4=\frac{1}{2}e^{i(k+k')a}\left(1-\frac{k}{k'}\right)$$

$$c_1+c_2-\frac{1}{2}e^{i(k-k')a}\left(1+\frac{k}{k'}\right)-\frac{1}{2}e^{i(k+k')a}\left(1-\frac{k}{k'}\right)=0$$

$$c_1-c_2-\frac{1}{2}\frac{k'}{k}e^{i(k-k')a}\left(1+\frac{k}{k'}\right)+\frac{1}{2}\frac{k'}{k}e^{i(k+k')a}\left(1-\frac{k}{k'}\right)=0$$

$$\cancel{2c_1-\frac{1}{2}e^{i(k-k')a}\left(1+\frac{k}{k'}\right)-\frac{1}{2}\frac{k'}{k}e^{i(k-k')a}\left(1+\frac{k}{k'}\right)=0}$$

$2c_2$

$$2c_1-\frac{1}{2}\left(1+\frac{k'}{k}\right)\left(1+\frac{k}{k'}\right)e^{i(k-k')a}-\frac{1}{2}\left(1-\frac{k'}{k}\right)\left(1-\frac{k}{k'}\right)e^{i(k+k')a}=0$$

$$2c_2-\frac{1}{2}\left(1-\frac{k'}{k}\right)\left(1+\frac{k}{k'}\right)e^{i(k-k')a}-\frac{1}{2}\left(1+\frac{k'}{k}\right)\left(1-\frac{k}{k'}\right)e^{i(k+k')a}=0$$

$$2c_1-\frac{1}{2}\left(1+\frac{k'}{k}+\frac{k}{k'}+1\right)e^{i(k-k')a}-\frac{1}{2}\left(1-\frac{k'}{k}-\frac{k}{k'}+1\right)e^{i(k+k')a}=0$$

$$2c_2-\frac{1}{2}\left(1-\frac{k'}{k}+\frac{k}{k'}-1\right)e^{i(k-k')a}-\frac{1}{2}\left(1-\frac{k}{k'}+\frac{k'}{k}-1\right)e^{i(k+k')a}=0$$

$$2c_1-\frac{1}{2}\left(2+\frac{k'^2+k^2}{kk'}\right)e^{i(k-k')a}-\frac{1}{2}\left(2-\frac{k'^2+k^2}{k'k}\right)e^{i(k+k')a}=0$$

$$2c_2+\frac{1}{2}\frac{k'^2-k^2}{k'k}e^{i(k-k')a}+\frac{1}{2}\frac{k^2-k'^2}{k'k}e^{i(k+k')a}=0$$

$$2c_1-e^{i(k-k')a}-e^{i(k+k')a}-\frac{k'^2+k^2}{2kk'}e^{i(k-k')a}+\frac{k'^2+k^2}{2k'k}e^{i(k'+k)a}=0$$

$$2c_2+\frac{1}{2}\frac{k'^2-k^2}{k'k}\left[e^{i(k-k')a}-e^{i(k+k')a}\right]=0$$

$$2c_1=e^{ika}\left[e^{-ik'a}+e^{ik'a}+\frac{k'^2+k^2}{2kk'}e^{-ik'a}-\frac{k'^2+k^2}{2kk'}e^{ik'a}\right]$$

$$2c_2=\frac{1}{2}\frac{k'^2-k^2}{k'k}e^{ika}\left[e^{ik'a}-e^{-ik'a}\right]$$

UKM T5-35

$$c_1 = \frac{e^{ika}}{2}\left[e^{-ik'a} + e^{ik'a} + \frac{k'^2+k^2}{2k'k}\left(-e^{ik'a} + e^{-ik'a}\right)\right]$$

$$c_2 = \frac{1}{4}\frac{k'^2-k^2}{k'k}e^{ika}\left(e^{ik'a} - e^{-ik'a}\right)$$

$$c_1 = e^{ika}\left[\cos(k'a) + i\frac{k'^2+k^2}{2k'k}\sin(k'a)\right]$$

$$c_2 = ie^{ika}\frac{k'^2-k^2}{2k'k}\sin(k'a)$$

$$R = \left|\frac{c_2}{c_1}\right|^2 = \frac{\left(\frac{k'^2-k^2}{2k'k}\right)^2\sin^2(k'a)}{\left[\cos(k'a) + i\frac{k'^2+k^2}{2k'k}\sin(k'a)\right]\left[\cos(k'a) - i\frac{k'^2+k^2}{2k'k}\sin(k'a)\right]}$$

$$= \frac{(k'^2-k^2)\sin^2(k'a)}{\left[4k^2k'^2\cos^2(k'a) + \left(k'^2+k^2\right)^2\sin^2(k'a)\right]}$$

$$T = \left|\frac{c_5}{c_1}\right|^2 = \frac{1}{\left[\cos(k'a) + i\frac{k'^2+k^2}{2k'k}\sin(k'a)\right]\left[\cos(k'a) - i\frac{k'^2+k^2}{2k'k}\sin(k'a)\right]}$$

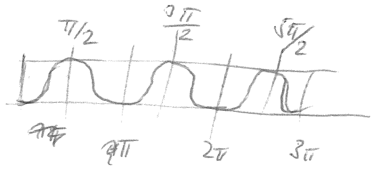
$$= \frac{4k'^2k^2}{4k'^2k^2\cos^2(k'a) + (k^2+k'^2)^2\sin^2(k'a)} = \frac{4k'^2k^2}{4k'^2k^2 + (k^2-k'^2)^2\sin^2(k'a)}$$

$1 - \sin^2$

$$\frac{R}{T} \overset{?}{=} \left|\frac{c_2}{c_5}\right|^2 = \frac{(k'^2-k^2)^2}{4k'^2k^2}\sin^2(k'a)$$

above step/well energy

$\sin^2$:

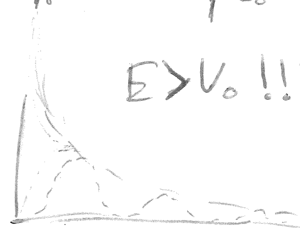


$$k' = \frac{\sqrt{2m(E_0-V)}}{\hbar} = k\cdot\sqrt{\frac{E_0-V}{E_0}}$$

$$= \frac{\left(2m(E_0-V) - 2mE_0\right)^2}{4\,2^2m^2(E_0-V)^2E_0^2}\sin^2(k'a) =$$

$$= \frac{(E_0-V-E_0)^2}{(E_0-V)E_0}\sin^2(k'a) = \frac{V^2}{(E_0-V)E_0}\sin^2(k'a)$$

$E > V_0$!!!

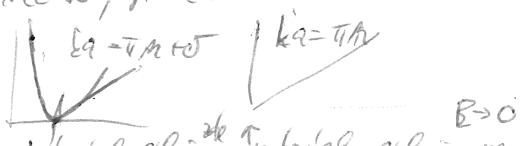


$V>0$ $$\lim_{E_0\to V+}\frac{V^2}{(E_0-V)E_0}\sin^2(k'a) = \lim_{E_0\to V+}\frac{V^2}{\delta V}\sin^2\left(\frac{\sqrt{2m(E_0-V)}}{\hbar}a\right) =$$

$$= \lim_{E_0\to V+}\frac{V}{\delta}\frac{2m(E_0-V)}{2\hbar}a^2 = \frac{V\,2ma^2}{2\hbar}$$ OK – výška bariéry · délka²

$V<0$ $$\lim_{E_0\to 0+}\frac{V^2}{(E_0-V)E_0}\sin^2(k'a) \to C\cdot\frac{1}{E_0}\sin^2(k'a)$$ OK – nekonečno, pokud $\sin(k'a)=0$

konečné



podmínka splněna ale podmínka splněna pro