UKM 2023
T 16.5. -1

H-F veta

1.1.
$$\frac{dE_\lambda}{d\lambda} = \frac{d}{d\lambda}\langle\psi|H_\lambda|\psi\rangle = \left(\frac{d\langle\psi|}{d\lambda}\right) H_\lambda|\psi\rangle + \langle\psi|H_\lambda \frac{d|\psi\rangle}{d\lambda}\rangle + \langle\psi|\left(\frac{dH_\lambda}{d\lambda}\right)|\psi\rangle$$

$$= \frac{d\langle\psi|}{d\lambda} E_\lambda|\psi\rangle + \langle\psi|E_\lambda \frac{d|\psi\rangle}{d\lambda} + \langle\psi|\frac{dH_\lambda}{d\lambda}|\psi\rangle$$

$$= E_\lambda \frac{d}{d\lambda}\underbrace{\langle\psi|\psi\rangle}_{1} + \langle\psi|\frac{dH_\lambda}{d\lambda}|\psi\rangle = \langle\psi|\frac{dH_\lambda}{d\lambda}|\psi\rangle$$

1.2. LHO:

$$E = \hbar\omega\left(n+\tfrac{1}{2}\right) \qquad H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$$

$$\frac{dE}{d\omega} = \hbar\left(n+\tfrac{1}{2}\right) \qquad \frac{dH}{d\omega} = m\omega x^2 = \frac{1}{2}m\omega\frac{\hbar}{m\omega}\left(a^2+a^{+2}+2a^+a+1\right)$$

1.3
$$x = \frac{\alpha}{\sqrt{2}}(a+a^+) \qquad x^2 = \frac{\alpha^2}{2}(a+a^+)(a+a^+) = \frac{\alpha^2}{2}\left(a^2+a^{+2}+2a^+a+1\right)$$

$$\frac{dE}{d\omega}: \langle 0|\hbar\left(\hat{n}+\tfrac{1}{2}\right)|0\rangle = \hbar\left(n+\tfrac{1}{2}\right)$$

$$\frac{dE}{d\omega}: \frac{1}{\sqrt{2}}\left(\langle 0|+\langle 1|\right)\hbar\left(\hat{n}+\tfrac{1}{2}\right)\left[\frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right)\right] = \cancel{\frac{\hbar}{2}\left(\langle 0|+\langle 1|\right)\left(|1\rangle\right) = \frac{\hbar}{2}}$$

$$= \frac{\hbar}{2}\left(\langle 0|+\langle 1|\right)\left[\tfrac{1}{2}|0\rangle + \tfrac{3}{2}|1\rangle\right] = \frac{\hbar}{2}\left(\tfrac{1}{2}+\tfrac{3}{2}\right) = \hbar$$

$$\frac{1}{\sqrt{2}}\left(\langle 0|+\langle 2|\right)\hbar\left(\hat{n}+\tfrac{1}{2}\right)\left[\frac{1}{\sqrt{2}}\left(|0\rangle+|2\rangle\right)\right] =$$

$$= \frac{\hbar}{2}\left(\langle 0|+\langle 2|\right)\left(\tfrac{1}{2}|0\rangle + \tfrac{5}{2}|2\rangle\right) = \frac{\hbar}{2}\left(\tfrac{1}{2}+\tfrac{5}{2}\right) = \frac{3}{2}\hbar$$

$$\frac{dH}{d\omega} = \frac{1}{2}\hbar\left(a^2+a^{+2}+2a^+a+1\right)$$

$$\langle 0|\tfrac{1}{2}\hbar\left(a^2+a^{+2}+2a^+a+1\right)|0\rangle = \tfrac{1}{2}\hbar\langle 0|\hat{1}|0\rangle = \tfrac{1}{2}\hbar$$

$$\frac{1}{\sqrt{2}}\left(\langle 0|+\langle 1|\right)\tfrac{1}{2}\hbar\left(a^2+a^{+2}+2a^+a+1\right)\frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right) = \frac{\hbar}{4}\left[\langle 0|1|0\rangle + \langle 1|2a^+a+1|1\rangle\right] = \frac{\hbar}{4}\left[1+3\right] = \hbar \quad \text{OK}$$

$$\frac{1}{\sqrt{2}}\left(\langle 0|+\langle 2|\right)\tfrac{1}{2}\hbar\left(a^2+a^{+2}+2a^+a+1\right)\frac{1}{\sqrt{2}}\left(|0\rangle+|2\rangle\right) = \frac{1}{4}\left[\langle 0|1|0\rangle + \langle 2|2a^+a+1|2\rangle + \langle 0|a^2|2\rangle + \langle 2|a^{+2}|0\rangle\right] =$$

$$= \frac{1}{4}\left[1+5+\sqrt{2}+\sqrt{2}\right] = \frac{3}{2}+\frac{\sqrt{2}}{2} \quad \text{not OK}$$

2.1. $\left\langle \frac{\partial A}{\partial t} \right\rangle = 0$

$\hat{A} = \hat{x}\hat{p}$ ← $\hat{x}$ ani $\hat{p}$ na čase nezávisejí

$\frac{d\langle A\rangle}{dt}$ ← mění se jen fáze, která se vyruší $e^{i\delta} \cdot e^{-i\delta} = 1$

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T16.5.-2.

2.2. $[p^2, xp] = ppxp - xppp =$

$[x, p] = i\hbar$

$= (ppx - pxp + pxp - xpp)p$

$= p[p, x]p + [p, x]pp$

$= -2i\hbar p^2$

$[x^2, xp] = xxxp - xpxx = x(xxp - xpx + xpx - pxx)$

$= x^2[x, p] + x[x, p]x = \cancel{2x}\ 2i\hbar x^2$

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

2.3 $[H, xp] = \frac{1}{2m}[p^2, xp] + \frac{1}{2}m\omega^2[x^2, xp] =$

$= \frac{1}{2m}(-2i\hbar p^2) + \frac{1}{2}m\omega^2\, 2i\hbar x^2$

$= -i\hbar\frac{p^2}{m} + i\hbar m\omega^2 x^2 = -i\hbar\left(\frac{p^2}{m} - m\omega^2 x^2\right)$

$x\frac{d\,\frac{1}{2}x^2}{dx} = \frac{x \cdot 2x}{2} = \cancel{2}x^2$

2.4. $p^{\cancel{*}} = \frac{\sqrt{\hbar m\omega}}{\sqrt{2}}i(a^+ - a)$

$p^2 = \frac{\hbar m\omega}{2}(-1)[a^{+2} + a^2 - a^+a - aa^+] = -\frac{\hbar m\omega}{2}[a^{+2} + a^2 - 2a^+a - 1]$

$\langle 0|p^2|0\rangle = \frac{\hbar m\omega}{2}\langle 0|2a^+a + 1|0\rangle = \frac{\hbar m\omega}{2}$ $\rightarrow \left\langle\frac{p^2}{2m}\right\rangle = \frac{\hbar\omega}{4}$

$\langle 0|x^2|0\rangle = \frac{\alpha^2}{2}\langle 0|a^2 + a^{+2} + 2a^+a + 1|0\rangle = \frac{\alpha^2}{2}$ $\langle \cancel{m}m\omega^2 x^2\rangle = \frac{\alpha^2}{2}m\omega^2$

$= \frac{\hbar}{2m\omega}\cdot m\omega^{2} = \frac{\hbar\omega}{2}$

$\frac{1}{2}(\langle 0| + \langle 2|)p^2(|0\rangle + |2\rangle) = \frac{1}{2}(\langle 0| + \langle 2|)\frac{\hbar m\omega}{2}(-a^{+2} - a^2 + 2a^+a + 1)(|0\rangle + |2\rangle)$

$= \frac{\hbar m\omega}{4}(\langle 0| - a^2|2\rangle - \langle 2|a^{+2}|0\rangle + \langle 2|2a^+a + 1|2\rangle + \langle 0|1|0\rangle)$

$= \frac{\hbar m\omega}{\cancel{2}4}(-\sqrt{2} - \sqrt{2} + 5 + 1) = \frac{3 \cancel{+} \sqrt{2}}{2}\hbar m\omega$

$\frac{1}{2}(\langle 0| + \langle 2|)\cancel{x}^2(|0\rangle + |2\rangle) = \frac{1}{2}(\langle 0| + \langle 2|)\frac{\alpha^2}{2}(a^2 + a^{+2} + 2a^+a + 1)(|2\rangle + |0\rangle)$

$= \frac{\alpha^2}{4}(\langle 0|a^{\cancel{+}2}|2\rangle + \langle 2|a^{+2}|0\rangle + \langle 2|2a^+a + 1|2\rangle + \langle 0|1|0\rangle)$

$= \frac{\alpha^2}{4}(\sqrt{2} + \sqrt{2} + 5 + 1) = \frac{3 + \sqrt{2}}{2}\alpha^2$

$2\left\langle\frac{p^2}{2m}\right\rangle = \left(\frac{3 - \sqrt{2}}{2}\hbar\omega\right) \neq \langle m\omega^2 x^2\rangle = \frac{3 + \sqrt{2}}{2}\frac{m\omega^2\hbar}{m\omega} = \frac{3 + \sqrt{2}}{2}\hbar\omega$