Plán: · doplňkové $A_3$, $A_{4s}$ OKM-2023-T4-1

· střední hodnoty čas $\rho$

$\frac{1}{\sqrt{2}}(|0\rangle + \phi|3\rangle) \to \int, |\rangle$, matrix

· $i\hbar$ ŠR

$$\mathbb{A} = \begin{pmatrix} -2i & & & & \\ & -i & & 0 & \\ & & 0 & & \\ & 0 & & i & \\ & & & & 2i \end{pmatrix} \quad \dots \quad \frac{\hat{d}}{d\phi} = \hat{\mathbb{A}}$$

$$f_{s_1}(\phi) = \frac{1}{\sqrt{\pi}} \sin(\phi) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{2i}\left(e^{i\phi} - e^{-i\phi}\right)\right) = \frac{1}{2i\sqrt{\pi}}\left(e^{i\phi} - e^{-i\phi}\right)$$

$$= \frac{1}{\sqrt{2}i}\left(\frac{1}{\sqrt{2\pi}} e^{i\phi} - \frac{1}{\sqrt{2\pi}} e^{-i\phi}\right) = \frac{1}{i\sqrt{2}}\left[f_1(\phi) - f_{-1}(\phi)\right]$$

$$|s_1\rangle = \frac{1}{i\sqrt{2}}\left(|1\rangle - |-1\rangle\right)$$

$$\mathbb{S}_1 = \frac{1}{i\sqrt{2}} \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{array}{l} \leftarrow n = -2 \\ \leftarrow n = -1 \\ \leftarrow n = 0 \\ \leftarrow n = 1 \\ \leftarrow n = 2 \end{array}$$

$$\mathbb{A} s_1 = \frac{1}{i\sqrt{2}} \begin{pmatrix} 0 \\ i \\ 0 \\ i \\ 0 \end{pmatrix} \to \text{není vl. vektor}$$

$$\mathbb{A}(\mathbb{A} s_1) = \frac{1}{i\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \frac{-1}{i\sqrt{2}} \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = -\mathbb{S}_1 \to \text{je vl. vektor}$$

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$\langle \hat{A} \rangle$ obecná střední hodnota, stav "míne"

$$\langle 0|\hat{A}|0\rangle = \frac{1}{2\pi}\int_0^{2\pi} 1 \frac{d}{d\phi} 1 \, d\phi = 0$$

$$\langle 1|\hat{A}|1\rangle = \frac{1}{2\pi}\int_0^{2\pi} e^{-i\phi} \frac{d}{d\phi} e^{i\phi} = i$$

+ bracket

$$+ \frac{1}{\pi}\int_0^{2\pi} \sin(\phi) \frac{d}{d\phi} \sin(\phi)\, d\phi = \frac{1}{\pi}\int_0^{2\pi} \sin(\phi)\cos(\phi) = 0$$

$\langle -1|\hat{A}|-1\rangle = -i$

· ověřit pomocí ket

$$\langle s_1|\hat{A}|s_1\rangle = \frac{1}{4(i)(-i)\pi}\int_0^{2\pi}\left(e^{-i\phi} - e^{i\phi}\right)\frac{d}{d\phi}\left(e^{i\phi} - e^{-i\phi}\right) d\phi$$

$$= +\frac{1}{4\pi}\int_0^{2\pi}\left(e^{-i\phi} - e^{+i\phi}\right)\left(i e^{i\phi} + i e^{-i\phi}\right) d\phi$$

$$= +\frac{1}{4\pi}\int_0^{2\pi}\left(i + i e^{-2i\phi} - i e^{2i\phi} - i\right) d\phi = 0 \quad \frac{1}{\sqrt{2}}\left[\,\cdot + \cdot\,\right]$$

$$\rho(\phi, t) = s_1^*(\phi, t)\, s_1(\phi, t); \qquad f_1(\phi, t) = \frac{1}{\sqrt{2\pi}} e^{i\phi} e^{-i\varepsilon t/\hbar}$$

$$= \frac{1}{+4\pi}\left(e^{-i\phi} e^{i\varepsilon t/\hbar} - e^{i\phi} e^{i\varepsilon t/\hbar}\right)\left(e^{i\phi} e^{-i\varepsilon t/\hbar} - e^{-i\phi} e^{-i\varepsilon t/\hbar}\right) =$$

$$= +\frac{1}{4\pi}\left(e^{-i\phi} - e^{+i\phi}\right)\left(e^{i\phi} - e^{-i\phi}\right) = \frac{1}{\pi}\left(\underbrace{\frac{1}{-2i}\left(e^{-i\phi} - e^{i\phi}\right)}_{-\sin(-\phi) = \sin(\phi)} \underbrace{\frac{1}{2i}\left(e^{i\phi} - e^{-i\phi}\right)}_{\sin(\phi)}\right)$$

$$= \frac{1}{\pi}\sin^2(\phi)$$

better to use $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ 2022/, OKMC_09_03

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$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\boxed{2}$

$\psi(\phi) = \frac{1}{2\sqrt{\pi}}(1 + e^{i\phi})$

R: $\langle\psi|\psi\rangle = \frac{1}{4\pi}\int_0^{2\pi}(1 + e^{-i\phi})(1 + e^{i\phi})\,d\phi = \frac{1}{4\pi}\int_0^{2\pi}[1 + e^{-i\phi}e^{i\phi}]\,d\phi = \frac{4\pi}{4\pi} = 1$ OK

$\langle\hat{A}\rangle = ?$

a) $\int_0^{2\pi}\psi^*(\phi)\,\hat{A}\,\psi(\phi)\,d\phi = \frac{1}{4\pi}\int_0^{2\pi}(1 + e^{-i\phi})\frac{d}{d\phi}(1 + e^{i\phi})\,d\phi$

$\boxed{3.2}$ $= \frac{1}{4\pi}\int_0^{2\pi}(1 + e^{-i\phi})(0 + i e^{i\phi})\,d\phi$

$= \frac{1}{4\pi}\int_0^{2\pi}(\dots\ \dots\ \dots\ i\cdot 1)\,d\phi = \frac{i}{2}$  $\hat{A}|n\rangle = in|n\rangle$

b) $\langle\psi|\hat{A}|\psi\rangle = \left[\frac{1}{\sqrt{2}}(\langle 0| + \langle 1|)\right]\hat{A}\left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right] = \frac{1}{2}[\langle 0| + \langle 1|][0\cdot|0\rangle + i|1\rangle]$

$= \frac{i}{2}$

c) $\psi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\1\\1\\0\end{pmatrix}$  $A = \begin{pmatrix}-2i & & & & 0\\ & -i & & & \\ & & 0 & & \\ & & & i & \\ 0 & & & & 2i\end{pmatrix}$  $\psi^* A\, \psi = \frac{1}{2}(0\,0\,1\,1\,0)\begin{pmatrix}-2i & & & & 0\\ & -i & & & \\ & & 0 & & \\ & & & i & \\ 0 & & & & 2i\end{pmatrix}\begin{pmatrix}0\\0\\1\\1\\0\end{pmatrix}$

$= \frac{1}{2}(0\,0\,1\,1\,0)\begin{pmatrix}0\\0\\0\\i\\0\end{pmatrix} = \frac{i}{2}$ OK

$\rho(\phi) = \psi^*(\phi)\,\psi(\phi)$

$\psi(\phi, t) = \sum_n f_n(\phi)\,e^{-i\varepsilon_n t/\hbar}$

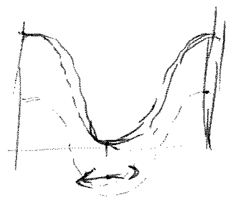
$\boxed{4.1}$ $\psi(\phi, t) = \frac{1}{2\sqrt{\pi}}(1\cdot\underbrace{e^0}_{=1} + e^{i\phi}e^{-i\varepsilon_1 t/\hbar})$  $\psi^*(\phi, t) = \frac{1}{2\sqrt{\pi}}(1 + e^{-i\phi}e^{i\varepsilon_1 t/\hbar})$

$\boxed{4.2}$ $\psi^*(\phi,t)\,\psi(\phi, t) = \frac{1}{4\pi}(1 + e^{i\phi}e^{-i\varepsilon_1 t/\hbar})(1 + e^{-i\phi}e^{i\varepsilon_1 t/\hbar})$

$= \frac{1}{4\pi}(1 + e^{-i\phi}e^{i\varepsilon_1 t/\hbar} + e^{i\phi}e^{-i\varepsilon_1 t/\hbar} + 1)$

$= \frac{1}{4\pi}[2 + 2\cos(\phi - \varepsilon_1 t/\hbar)]$

$= \frac{1}{2\pi}[1 + \cos(\phi - \varepsilon_1 t/\hbar)]$



$\psi$ $\langle\hat{O}\rangle = \sum_n |c_n|^2\, O_{nn}$

$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2\pi}}\,\mathbb{1} + \frac{1}{\sqrt{2\pi}} e^{i\phi}\right)$ Ckal-2023 T4-3

$= \frac{1}{2\sqrt{\pi}}\left(1 + e^{i\phi}\right)$

$\boxed{3.3}$ $\hat{B} = e^{i\phi}$

$\langle\psi|\mathbb{B}|\psi\rangle$

a) $\int_0^{2\pi} \psi^*(\phi)\, e^{i\phi}\, \psi(\phi)\, d\phi = \frac{1}{4\pi}\int_0^{2\pi} (1 + e^{-i\phi})\, e^{i\phi}(1 + e^{i\phi})\, d\phi$

$= \frac{1}{4\pi}\int_0^{2\pi}\left[e^{i\phi} + e^{2i\phi} + 1 + e^{i\phi}\right] d\phi = \underline{\underline{\frac{1}{2}}}$

$\hat{B}|n\rangle = |n+1\rangle$

b) $\frac{1}{\sqrt{2}}\left(\langle 0| + \langle 1|\right)\hat{B}\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = \frac{1}{2}\left(\langle 0| + \langle 1|\right)\left(|1\rangle + |2\rangle\right) = \underline{\underline{\frac{1}{2}}}$

c) $$\mathbb{B} = \begin{array}{c} \begin{matrix} -2 & & & & 2 \end{matrix} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array} \qquad \psi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\psi^* \mathbb{B}\, \psi = \frac{1}{\sqrt{2}}\left(0\,0\,1\,1\,0\right)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ \cancel{0} \\ 1 \\ 1 \end{pmatrix} = \underline{\underline{\frac{1}{2}}}$$

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5.1 $\psi(x,t) = \sum_n c_n\, \nu_n(x)\, e^{-\frac{i\varepsilon_n t}{\hbar}}$

$t \to -i\tau$

$\psi(x,\tau) = \sum_n c_n\, \nu_n(x)\, e^{-\frac{i\varepsilon_n}{\hbar}(-i\tau)}$

$= \sum_n c_n\, \nu_n(x)\, e^{-\varepsilon_n \tau/\hbar}$

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$A = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \left|A - \mathbb{1}\lambda\right| = \begin{vmatrix} -\lambda & 1/2 \\ 1/2 & -\lambda \end{vmatrix} \to |\;| = (\lambda^2 - \frac{1}{4})$

$\Rightarrow \lambda = \pm\frac{1}{2}$