plain - ψ_2 of LHO, OG to $|a\rangle$

a, a^+ action on states, $[a, a^+]$, exp values $\langle x^2 \rangle, \dots$ $\langle p \rangle, \langle p^2 \rangle$

a, a^+ matrix repre, x, x^2, p^2 repre

LHO in E field!

$$\psi_0 = \frac{1}{\sqrt{\alpha\sqrt{\pi}}} e^{-\frac{x^2}{2\alpha^2}} \qquad \psi_1 = \sqrt{\frac{2}{\alpha\sqrt{\pi}}} \frac{x}{\alpha} e^{-\frac{x^2}{2\alpha^2}} \qquad \psi_2 = \frac{1}{\sqrt{2\alpha\sqrt{\pi}}} \left(2\frac{x^2}{\alpha^2} - 1\right) e^{-\frac{x^2}{2\alpha^2}}$$

$$\langle \psi_2 | \psi_2 \rangle = \frac{1}{2\alpha\sqrt{\pi}} \int_{-\infty}^{\infty} \left(2\frac{x^2}{\alpha^2} - 1\right)^2 e^{-\frac{x^2}{\alpha^2}} dx = \frac{1}{2\alpha\sqrt{\pi}} \int_{-\infty}^{\infty} \left[4\frac{x^4}{\alpha^4} - 4\frac{x^2}{\alpha^2} + 1\right] e^{-\frac{x^2}{\alpha^2}} dx$$

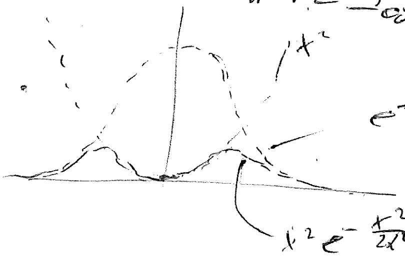
$$\int e^{-\frac{x^2}{\alpha^2}} = \alpha\sqrt{\pi} \quad \Big| \quad \int x^2 e^{-\frac{x^2}{\alpha^2}} = \frac{\alpha^2}{2} \alpha\sqrt{\pi} \quad \Big| \quad \int x^4 e^{-\frac{x^2}{\alpha^2}} = \frac{3\alpha^2}{2} \frac{\alpha^2}{2} \alpha\sqrt{\pi}$$

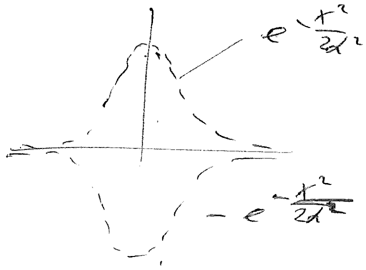
$$= \frac{1}{2\alpha\sqrt{\pi}} \alpha\sqrt{\pi} \left[4 \cdot \frac{3\alpha^4}{4\alpha^4} - 4 \cdot \frac{\alpha^2}{2\alpha^2} + 1\right] = \frac{1}{2} \cdot [3 - 2 + 1] = 1 \quad \text{OK}$$

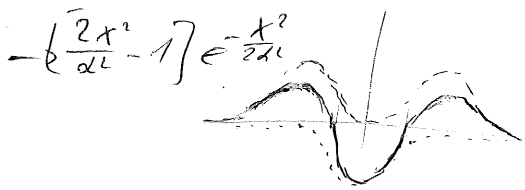
$$\langle \psi_2 | \psi_0 \rangle = \frac{1}{\sqrt{\alpha\sqrt{\pi}}} \frac{1}{\sqrt{2\alpha\sqrt{\pi}}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\alpha^2}} \left(2\frac{x^2}{\alpha^2} - 1\right) e^{-\frac{x^2}{2\alpha^2}} dx$$

$$= \frac{1}{\alpha\sqrt{\pi}\sqrt{2}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{\alpha^2}} \left(2\frac{x^2}{\alpha^2} - 1\right) dx = \frac{1}{\alpha\sqrt{\pi}\sqrt{2}} \left[\frac{2\alpha^2}{2\alpha^2} - 1\right] \alpha\sqrt{\pi} = \frac{1}{\sqrt{2}}[1-1] = 0$$

OK







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a, a^+

$$\hat{a} = \frac{1}{\sqrt{2}} \sqrt{\frac{m\omega}{\hbar}} \left(\hat{x} + \frac{i\hat{p}}{m\omega}\right) \qquad \hat{a}^+ = \frac{1}{\sqrt{2}} \sqrt{\frac{m\omega}{\hbar}} \left(\hat{x} - \frac{i\hat{p}}{m\omega}\right)$$

interesting as $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$

$\hat{a}^+|n\rangle = \sqrt{n+1}|n+1\rangle$

• $a^+a|n\rangle = a^+\sqrt{n}|n-1\rangle = \sqrt{n}\, a^+|n-1\rangle = \sqrt{n}\sqrt{n}|n\rangle = n|n\rangle$

$$a^+a = \frac{1}{\sqrt{2}}\sqrt{\frac{m\omega}{\hbar}}\left(x - \frac{ip}{m\omega}\right)\frac{1}{\sqrt{2}}\sqrt{\frac{m\omega}{\hbar}}\left(x + \frac{ip}{m\omega}\right) =$$

$$= \frac{1}{2}\frac{m\omega}{\hbar}\left[x^2 + \frac{ixp}{m\omega} - \frac{ipx}{m\omega} + \frac{p^2}{m^2\omega^2}\right]$$

$$= \frac{1}{2}\frac{m\omega}{\hbar}x^2 + \frac{1}{2}\frac{i}{\hbar}[xp - px] + \frac{1}{2}\frac{p^2}{\hbar m\omega}$$

$[x,p]f = [x, -i\hbar\frac{d}{dx}]f = -i\hbar x\frac{df}{dx} + i\hbar\frac{d}{dx}(xf) = i\hbar f$

$[x,p] = i\hbar$

$$= \frac{1}{2}\frac{m\omega}{\hbar}x^2 + \frac{1}{2}\frac{i}{\hbar}i\hbar + \frac{1}{2}\frac{p^2}{\hbar m\omega} = \frac{1}{2}\frac{m\omega}{\hbar}x^2 - \frac{1}{2} + \frac{1}{2}\frac{p^2}{\hbar m\omega}$$

$$\frac{\hbar\omega}{\hbar\omega} \to \frac{1}{\hbar\omega}\left[m\omega^2x^2 - \frac{1}{2}\hbar\omega + \frac{1}{2}\frac{p^2}{m}\right] = \frac{1}{\hbar\omega}\left[\hat{H} - \frac{1}{2}\hbar\omega\right]$$

$$\Rightarrow \hat{H} = \hbar\omega a^+a + \frac{1}{2}\hbar\omega = \hbar\omega\left[a^+a + \frac{1}{2}\right]$$

$$\sim \hbar\omega\left[n + \frac{1}{2}\right]$$

$a, a^+ \to x, p$

$$a + a^+ = \sqrt{2}\sqrt{\frac{m\omega}{\hbar}}\hat{x} \to \hat{x} = \sqrt{\frac{\hbar}{m\omega}}\frac{1}{\sqrt{2}}(a + a^+)$$

$$a - a^+ = \sqrt{2}\sqrt{\frac{m\omega}{\hbar}}\frac{i}{m\omega}p \to p = \frac{1}{\sqrt{2}}\sqrt{\hbar m\omega}\,(-i)(a - a^+)$$

$$\to p = \frac{i}{\sqrt{2}}\sqrt{\hbar m\omega}\,(a^+ - a)$$

pomocí $[x,p] = i\hbar$

$$[a, a^+] = \frac{1}{2}\frac{m\omega}{\hbar}\left[x + \frac{ip}{m\omega}, x - \frac{ip}{m\omega}\right] = \frac{1}{2}\frac{m\omega}{\hbar}\left[\left[x, -\frac{ip}{m\omega}\right] + \left[\frac{ip}{m\omega}, x\right]\right]$$

$$= \frac{i}{\hbar}[p, x] = \frac{i}{\hbar}(-i\hbar) = 1 \qquad \to aa^+ - a^+a = 1$$

$aa^+ = 1 + a^+a$

$$x^2 = \left[\sqrt{\frac{\hbar}{m\omega}}\frac{1}{\sqrt{2}}(a + a^+)\right]^2 = \frac{\hbar}{2m\omega}(a + a^+)(a + a^+) = \frac{\hbar}{2m\omega}\left[aa + a^+a^+ + a^+a + aa^+\right]$$

$$= \frac{\hbar}{2m\omega}\left[a^2 + a^{+2}\ 2a^+a + 1\right]$$

$a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$ $\qquad a|n\rangle = \sqrt{n}\,|n-1\rangle$ $\qquad$ 2022-UKM-TB

$$a^{*} = \frac{1}{\sqrt{2}}\underbrace{\sqrt{\frac{m\omega}{\hbar}}}_{\frac{1}{\alpha}}\left(x + \frac{i}{m\omega}\underbrace{p}_{-i\hbar\frac{d}{dx}}\right) \qquad |0\rangle \doteq \frac{1}{\sqrt{\alpha\sqrt{\pi}}}\, e^{-\frac{x^2}{2\alpha^2}}$$

$$a^+ = \frac{1}{\sqrt{2}}\frac{x}{\alpha} - \frac{\alpha}{\sqrt{2}}\frac{d}{dx}$$

$$= \frac{1}{\sqrt{2}}\frac{1}{\alpha}\Big(x + \underbrace{\frac{\hbar}{m\omega}}_{\alpha^2}\frac{d}{dx}\Big) = \frac{1}{\sqrt{2}}\left(\frac{1}{\alpha}x + \alpha^2\frac{d}{dx}\right) = \frac{1}{\sqrt{2}}\frac{x}{\alpha} + \frac{\alpha}{\sqrt{2}}\frac{d}{dx}$$

$$\left[\frac{1}{\sqrt{2}}\frac{x}{\alpha} + \frac{\alpha}{\sqrt{2}}\frac{d}{dx}\right]\frac{1}{\sqrt{\alpha\sqrt{\pi}}}\, e^{-\frac{x^2}{2\alpha^2}} = \left[\frac{1}{\sqrt{2}\alpha}\, x\, e^{-\frac{x^2}{2\alpha^2}} + \frac{\alpha}{\sqrt{2}}\left(-\frac{x}{\alpha^2}\right)e^{-\frac{x^2}{2\alpha^2}}\right]\frac{1}{\sqrt{\alpha\sqrt{\pi}}} =$$

$$= \frac{1}{\sqrt{\alpha\sqrt{\pi}}}\left\{ e^{-\frac{x^2}{2\alpha^2}}\left[\frac{x}{\sqrt{2}\alpha} - \frac{x}{\sqrt{2}\alpha}\right] = 0\right.$$

$$a^{*} \to \frac{1}{\sqrt{2}}\frac{x}{\alpha} + \frac{\alpha}{\sqrt{2}}\frac{d}{dx}$$

$$a^+ \to \frac{1}{\sqrt{2}}\frac{x}{\alpha} - \frac{\alpha}{\sqrt{2}}\frac{d}{dx} \qquad a^+|0\rangle = \frac{1}{\sqrt{\alpha\sqrt{\pi}}}\, e^{-\frac{x^2}{2\alpha^2}}\left[\frac{x}{\sqrt{2}\alpha} + \frac{x}{\sqrt{2}\alpha}\right]$$

$$= \frac{1}{\sqrt{\alpha\sqrt{\pi}}}\,\frac{\sqrt{2}\,x}{\alpha}\, e^{-\frac{x^2}{2\alpha^2}} \qquad \text{OK}$$

$$a^+|1\rangle = \left[\frac{1}{\sqrt{2}}\frac{x}{\alpha} - \frac{\alpha}{\sqrt{2}}\frac{d}{dx}\right]\frac{1}{\sqrt{\alpha\sqrt{\pi}}}\,\frac{\cancel{\sqrt{2}}\,x}{\cancel{\alpha}}\, e^{-\frac{x^2}{2\alpha^2}}$$

$$= \frac{1}{\sqrt{\alpha\sqrt{\pi}}}\,\frac{x^2}{\alpha^2}\, e^{-\frac{x^2}{2\alpha^2}} - \frac{1}{\sqrt{\alpha\sqrt{\pi}}}\left[e^{-\frac{x^2}{2\alpha^2}} + x\left(-\frac{x}{\alpha^2}\right)e^{-\frac{x^2}{2\alpha^2}}\right]$$

opposite sign with a

$$= \frac{1}{\sqrt{\alpha\sqrt{\pi}}}\, e^{-\frac{x^2}{2\alpha^2}}\left[\frac{x^2}{\alpha^2} - 1 + \frac{x^2}{\alpha^2}\right] = \frac{1}{\sqrt{\alpha\sqrt{\pi}}}\left[\frac{2x^2}{\alpha^2} - 1\right]e^{-\frac{x^2}{2\alpha^2}} \qquad \text{OK?} \quad \frac{1}{\sqrt{2}}\,!?$$

! $a^+|1\rangle \neq |2\rangle$ ale $a^+|1\rangle = \sqrt{2}\,|2\rangle$ $\quad \sqrt{2}\,|2\rangle$

$$|2\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{\alpha\sqrt{\pi}}}\left[\frac{2x^2}{\alpha^2} - 1\right]e^{-\frac{x^2}{2\alpha^2}}$$