UKM - 2021
III příprava

$$\langle p|q\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{-\frac{i}{\hbar}pq}$$

$$\langle p|x\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{-\frac{i}{\hbar}px}$$

$$|\psi\rangle \rightarrow \mathbb{1}|\psi\rangle = \int dx\, |x\rangle \underbrace{\langle x|\psi\rangle}_{\psi(x)} = \int dx\, \psi(x) |x\rangle \quad [\text{cf. } \textstyle\sum_i c_i |i\rangle]$$

index

$\psi(x) \leftarrow$ "jiné" koeficienty / spojité koeficienty...

$$\rightarrow \int dp\, |p\rangle \langle p|\psi\rangle$$

$$= \int dp\, |p\rangle \langle p| \int dx\, |x\rangle\langle x|\psi\rangle$$

$$= \int dp\, |p\rangle \int dx\, \underbrace{\langle p|x\rangle}_{\frac{1}{\sqrt{2\pi\hbar}} e^{-\frac{i}{\hbar}px}} \langle x|\psi\rangle$$

$$= \int dp\, |p\rangle \underbrace{\int dx \frac{1}{\sqrt{2\pi\hbar}} e^{-\frac{i}{\hbar}px}\, \psi(x)}_{?\ \psi(p)}$$

$$= \int dp\, |p\rangle\, \psi(p)$$

$$\rightarrow \psi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int dx\, e^{-\frac{i}{\hbar}px} \psi(x) \quad \rightarrow \text{Gauss}, \langle x\rangle \langle x^2\rangle \ldots$$

$$|0\rangle = \frac{1}{\sqrt[4]{\pi}} e^{-\frac{x^2}{2}}, \quad \text{vrátit } \hbar, m, \omega \rightarrow \frac{1}{\sqrt[4]{\pi}\sqrt{\alpha}} e^{-\frac{x^2}{2\alpha^2}} \qquad \alpha = \sqrt{\frac{\hbar}{m\omega}}$$

$$\psi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int dx\, e^{-\frac{i}{\hbar}px} \frac{1}{\sqrt[4]{\pi}\sqrt{\alpha}} e^{-\frac{x^2}{2\alpha^2}}$$

$$= \frac{1}{\sqrt{2\pi\hbar}\sqrt{\alpha}\sqrt[4]{\pi}} \int dx\, e^{-\frac{1}{2\alpha^2}\left(x^2 + \frac{2\alpha^2 i}{\hbar}px\right)}$$

$$x^2 + \frac{2i\alpha^2}{\hbar}px$$

$$A = 1 \quad B = \frac{i\alpha^2}{\hbar}p \quad B = -\frac{\alpha^4}{\hbar^2}p^2$$

$$= \frac{1}{\sqrt{2\pi\hbar}\sqrt{\alpha}\sqrt[4]{\pi}} \int dx\, e^{-\frac{1}{2\alpha^2}\left(x + \frac{i\alpha^2 p}{\hbar}\right)^2} e^{-\frac{1}{2\alpha^2}\frac{\alpha^4}{\hbar^2}p^2}$$

$$= \frac{1}{\sqrt{\alpha}\sqrt[4]{\pi}\sqrt{2\pi\hbar}} \underbrace{\int dx'\, e^{-\frac{x'^2}{2\alpha^2}}}_{\sqrt{\ }\ A = \frac{1}{2\alpha^2}},\ e^{-\frac{\alpha^2 p^2}{2\hbar^2}} \qquad \int e^{-Ax^2} = \sqrt{\frac{\pi}{A}}$$

$$= \sqrt{\pi \cdot 2\alpha^2} = \alpha\sqrt{2\pi}$$

$$= \frac{\alpha \sqrt{2\pi}}{\sqrt{\alpha}\sqrt[4]{\pi}\sqrt{2\pi\hbar}} e^{-\frac{\alpha^2 p^2}{2\hbar^2}}$$

$$= \frac{\sqrt{\alpha}}{\sqrt[4]{\pi}\sqrt{\hbar}} e^{-\frac{\alpha^2 p^2}{2\hbar^2}}$$

UKM 2021

UKM-2021
příprava IV

$$\psi(p) = \sqrt{\frac{\alpha}{\hbar\sqrt{\pi}}}\, e^{-\frac{\alpha^2 p^2}{2\hbar^2}}$$

Normalizace

$$\langle\psi|\psi\rangle = \frac{\alpha}{\hbar\sqrt{\pi}} \int_{-\infty}^{\infty} dp\, e^{-\frac{\alpha^2 p^2}{\hbar^2}}$$

$$\int e^{-Ax^2} = \sqrt{\frac{\pi}{A}} \qquad A = \frac{\alpha^2}{\hbar^2}$$

$$= \frac{\alpha}{\hbar\sqrt{\pi}} \sqrt{\frac{\pi \hbar^2}{\alpha^2}} = 1 \text{ ok}$$

UKM 2021
příprava

$$p = -i\hbar \frac{d}{dx}$$

$$\psi(x) = \frac{1}{\sqrt{\alpha}\sqrt[4]{\pi}}\, e^{-\frac{x^2}{2\alpha^2}} \qquad \psi(p) = \frac{\sqrt{\alpha}}{\sqrt{\hbar}\sqrt[4]{\pi}}\, e^{-\frac{\alpha^2 p^2}{2\hbar^2}}$$

$$\langle\psi(x)|p|\psi(x)\rangle = \int_{-\infty}^{\infty} dx \frac{1}{\sqrt{\alpha}\sqrt[4]{\pi}} e^{-\frac{x^2}{2\alpha^2}} \left(-i\hbar\frac{d}{dx}\right) \frac{1}{\sqrt{\alpha}\sqrt[4]{\pi}} e^{-\frac{x^2}{2\alpha^2}}$$

tak to se do nezajeď B

$= 0$ z nějakého důvodu

$$\int x^2 e^{-Ax^2} \Rightarrow \dots \frac{3}{2A}\frac{1}{2A}\sqrt{\frac{\pi}{A}}$$

$x^2 e^{-x^2 A}$

$$\langle\psi(p)|p|\psi(p)\rangle = 0$$

$$\langle\psi(p)|p^2|\psi(p)\rangle = \frac{\alpha}{\hbar\sqrt{\pi}} \int_{-\infty}^{\infty} p^2 e^{-\frac{\alpha^2 p^2}{\hbar^2}} dp \qquad A = \frac{\alpha^2}{\hbar^2}$$

$$= \frac{\alpha}{\hbar\sqrt{\pi}} \sqrt{\frac{\pi\hbar^2}{\alpha^2}}\, \frac{\hbar^2}{2\alpha^2} = \frac{\hbar^2}{2\alpha^2} \checkmark$$

$$\langle\psi(x)|p^2|\psi(x)\rangle = \frac{(-i\hbar)^2}{\alpha\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\alpha^2}} \frac{d^2}{dx^2} e^{-\frac{x^2}{2\alpha^2}} dx$$

$$= \frac{-\hbar^2}{\alpha\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\alpha^2}} \frac{d}{dx}\left(-\frac{2x}{2\alpha^2} e^{-\frac{x^2}{2\alpha^2}}\right) dx$$

$$= \frac{-\hbar^2}{\alpha\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\alpha^2}} \left(-\frac{1}{\alpha^2} + \frac{x^2}{\alpha^4}\right) e^{-\frac{x^2}{2\alpha^2}} dx$$

$$= \frac{\hbar^2}{\alpha\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{\alpha^2}} \left(\frac{1}{\alpha^2} - \frac{x^2}{\alpha^4}\right) dx$$

$$\sqrt{\pi}\,\alpha \qquad \alpha\sqrt{\pi}\,\frac{\alpha^2}{2}$$

$$= \frac{\hbar^2}{\alpha\sqrt{\pi}}\, \alpha\sqrt{\pi}\left[\frac{1}{\alpha^2} - \frac{1}{2\alpha^2}\right] = \frac{\hbar^2}{2\alpha^2} \checkmark \text{ OK}$$

UKM 202-

III - přípra

D • A: $|\uparrow\rangle, |\downarrow\rangle$

B: $|\uparrow\rangle, |\downarrow\rangle$

celková báze: $|\uparrow\rangle_A \otimes |\uparrow\rangle_B, \; |\uparrow\rangle_A \otimes |\downarrow\rangle_B, \; |\downarrow\rangle_A \otimes |\uparrow\rangle_B, \; |\downarrow\rangle_A \otimes |\downarrow\rangle_B$

zjednodušeně: $|\uparrow\uparrow\rangle, \; |\uparrow\downarrow\rangle, \; |\downarrow\uparrow\rangle, \; |\downarrow\downarrow\rangle$

D Víme $S_z^A |\uparrow_A\rangle_A = \frac{1}{2} |\uparrow_A\rangle_A$

$$S_z = S_z^A \otimes 1 + 1 \otimes S_z^B$$

$$S_z |\uparrow\rangle_A \otimes |\uparrow\rangle_B = S_z^A |\uparrow\rangle_A \otimes 1 |\uparrow\rangle_B + 1 |\uparrow\rangle_A \otimes S_z^B |\uparrow\rangle_B =$$

$$= \frac{1}{2} |\uparrow\uparrow\rangle + \frac{1}{2} |\uparrow\uparrow\rangle = |\uparrow\uparrow\rangle \; ; \; \underline{\langle\uparrow\uparrow|S_z|\uparrow\uparrow\rangle = 1}$$

→ zachová vlastní stav B → diagonální matice

$$S_z |\uparrow\downarrow\rangle = \frac{1}{2} |\uparrow\downarrow\rangle - \frac{1}{2} |\uparrow\downarrow\rangle = 0$$

$$S_z |\downarrow\uparrow\rangle = 0 \rightarrow \underline{\langle\uparrow\downarrow|S_z|\uparrow\downarrow\rangle = 0}$$

$$S_z |\downarrow\downarrow\rangle = -|\downarrow\downarrow\rangle \rightarrow \underline{\langle\downarrow\downarrow|S_z|\downarrow\downarrow\rangle = -1}$$

$$S_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \leftarrow \text{matice } S_z = S_z^A + S_z^B$$

$$S_z = S_z^A \otimes 1 + 1 \otimes S_z^B$$

$$\begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$

$$\begin{pmatrix} 1/2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & 0 \cdot \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ 0 \cdot \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & -1/2 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix} \qquad \begin{pmatrix} 1 \cdot \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} & 0 \cdot (0) \\ 0 \cdot \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & 1 \cdot \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \end{pmatrix}$$

$$\begin{pmatrix} 1/2 & & & 0 \\ & 1/2 & & \\ & & -1/2 & \\ 0 & & & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & & & 0 \\ & -1/2 & & \\ & & 1/2 & \\ 0 & & & -1/2 \end{pmatrix} = \begin{pmatrix} 1 & & & 0 \\ & 0 & & \\ & & 0 & \\ 0 & & & -1 \end{pmatrix}$$

UKM 2021
III příprava

$S_z^A \otimes S_z^B \to |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle$
$|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle$ jsou stále vlastní stavy

$$S_z^A \otimes S_z^B |\uparrow\uparrow\rangle = \tfrac{1}{2} \cdot \tfrac{1}{2} |\uparrow\uparrow\rangle = \tfrac{1}{4} |\uparrow\uparrow\rangle$$
$$S_z^A \otimes S_z^B |\uparrow\downarrow\rangle = \tfrac{1}{2} \cdot (-\tfrac{1}{2}) |\uparrow\downarrow\rangle = -\tfrac{1}{4} |\uparrow\downarrow\rangle$$
$$S_z^A \otimes S_z^B |\downarrow\uparrow\rangle = (-\tfrac{1}{2})(\tfrac{1}{2}) |\downarrow\uparrow\rangle = -\tfrac{1}{4} |\downarrow\uparrow\rangle$$
$$S_z^A \otimes S_z^B |\downarrow\downarrow\rangle = (-\tfrac{1}{2})(-\tfrac{1}{2}) |\downarrow\downarrow\rangle = \tfrac{1}{4} |\downarrow\downarrow\rangle$$

$$\to \begin{pmatrix} 1/4 & & & \varnothing \\ & -1/4 & & \\ & & -1/4 & \\ \varnothing & & & 1/4 \end{pmatrix}$$

$$\begin{pmatrix} 1/2 & \varnothing \\ \varnothing & -1/2 \end{pmatrix} \otimes \begin{pmatrix} 1/2 & \varnothing \\ \varnothing & -1/2 \end{pmatrix} = \begin{pmatrix} \tfrac{1}{2} \cdot \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} & \varnothing \\ \varnothing & -\tfrac{1}{2} \cdot \begin{pmatrix} 1/2 & \varnothing \\ 0 & -1/2 \end{pmatrix} \end{pmatrix} =$$

$$M = \sum_i S_i^A \otimes S_i^B$$
$$= S_x^A \otimes S_x^B + S_y^A \otimes S_y^B + S_z^A \otimes S_z^B$$

$$\tfrac{1}{4} \begin{pmatrix} \varnothing & 1 \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ 1 \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \varnothing \end{pmatrix} + \tfrac{1}{4} \begin{pmatrix} \varnothing & -i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ i \begin{pmatrix} 0 & -i \\ i & \varnothing \end{pmatrix} & \varnothing \end{pmatrix} + \tfrac{1}{4} \begin{pmatrix} 1 & & & \\ & -1 & & \varnothing \\ & \varnothing & -1 & \\ & & & 1 \end{pmatrix}$$

$$= \tfrac{1}{4} \begin{pmatrix} \varnothing & \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} \\ \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} & \varnothing \end{pmatrix} + \tfrac{1}{4} \begin{pmatrix} \varnothing & \begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix} \\ \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} & \varnothing \end{pmatrix} + \tfrac{1}{4} \begin{pmatrix} 1 & & & \\ & -1 & & \varnothing \\ & \varnothing & -1 & \\ & & & 1 \end{pmatrix}$$

$$= \tfrac{1}{4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle$ stále vlastní stavy
$|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$ se míchají $\pm$