volná částice, bariéra,
δ-potenciál, čas. závislost ...

- $\left(\frac{p^2}{2m} + V(x)\right)\psi(x) = E\,\psi(x)$ $\quad$ (x - repre.., $p = -i\hbar\frac{\partial}{\partial x}$)

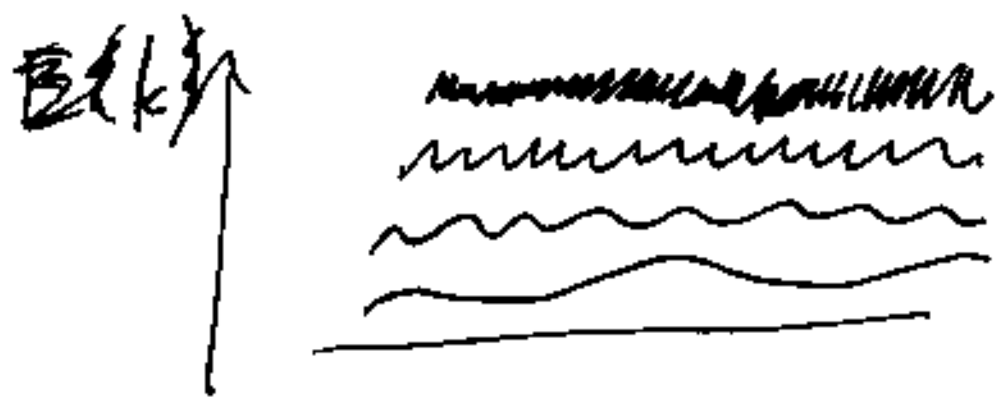
- volná částice $V(x) = \text{const}$, BÚNO : $V(x) = 0$
  tedy ψ se nezmění

úplně volná částice → E je vlastní parametr

$$\to \quad \frac{p^2}{2m}\psi(x) = E\,\psi(x)$$

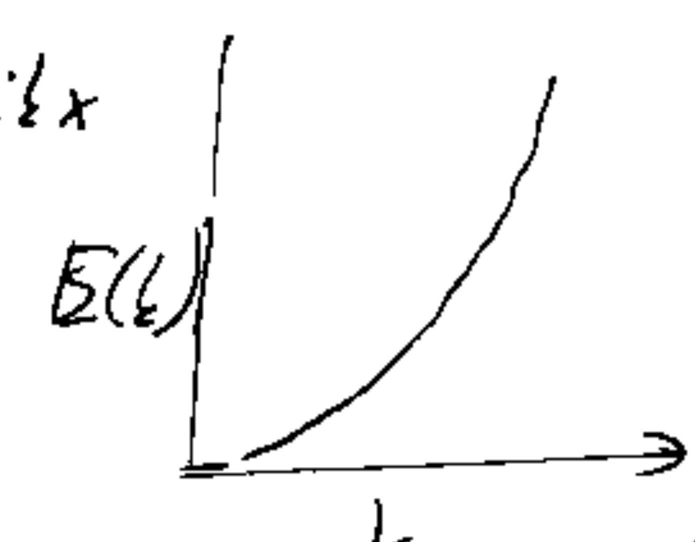
$$\Rightarrow \; -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\,\psi(x)$$

uhodneme e^{ikx} $\to -\frac{\hbar^2}{2m}\frac{\partial^2 e^{ikx}}{\partial x^2} = E\,e^{ikx}$

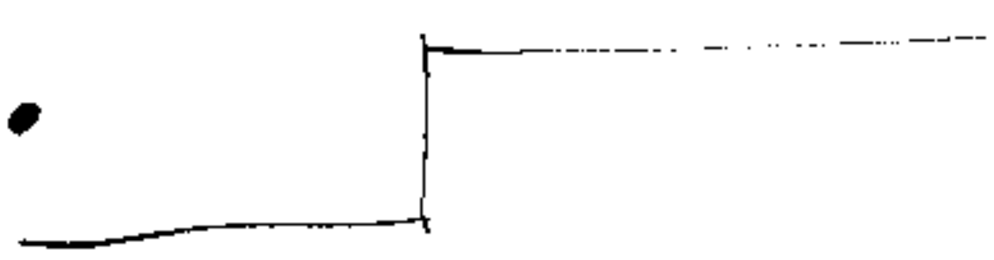


$$\frac{\hbar^2 k^2}{2m} e^{ikx} = E\,e^{ikx}$$

$$\Rightarrow E = \frac{\hbar^2 k^2}{2m}$$



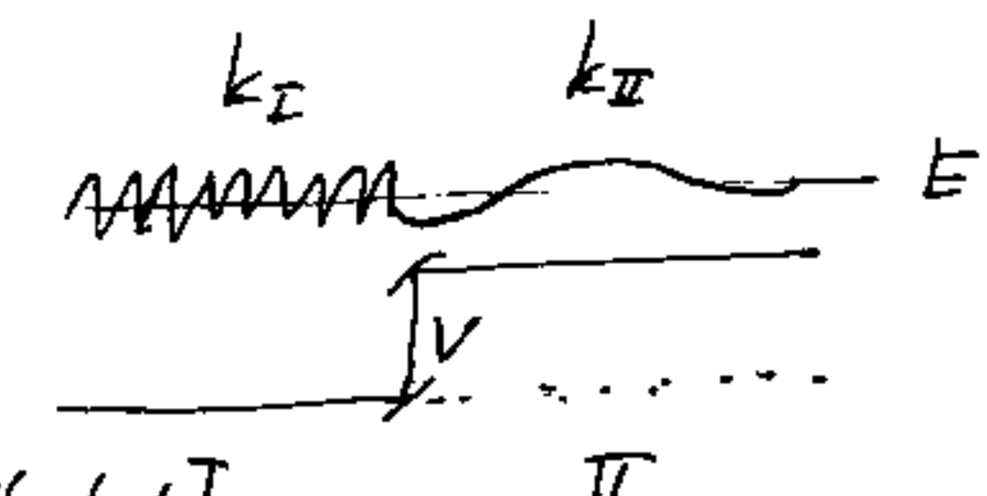
"disperzní relace"
(viďte Fyz IV)

- 

$$\frac{p^2}{2m}\psi + V\{\psi\} = E\psi$$

$$\frac{p^2}{2m}\psi + \cancel{(V-E)\psi = 0} = (E-V)\psi$$

posun



$E > V \Rightarrow$ změna k → je třeba řešit sešívání

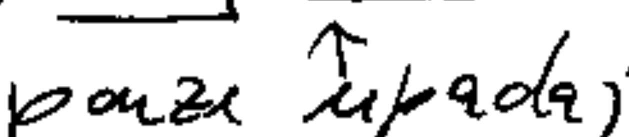
→ ups, co teď?

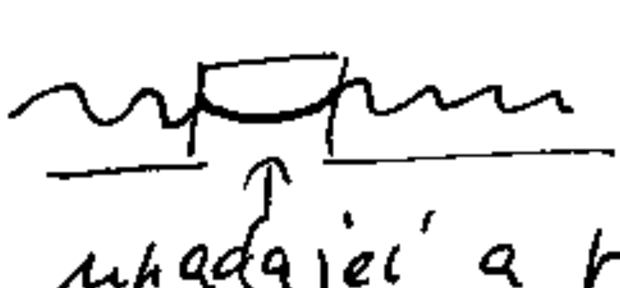
$$\frac{p^2}{2m}\psi = \underbrace{(E-V)}_{<0\;\to\;-\mathcal{E}}\psi$$

$$\frac{p^2}{2m}\psi = -\mathcal{E}\psi$$

$$-\frac{\hbar^2\partial^2\psi}{2m\,\partial x^2} = -\mathcal{E}\psi$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = \mathcal{E}\psi$$

$\to e^{\pm kx}$; pokud je bariéra  pouze upadající řešení



upadající a rostoucí

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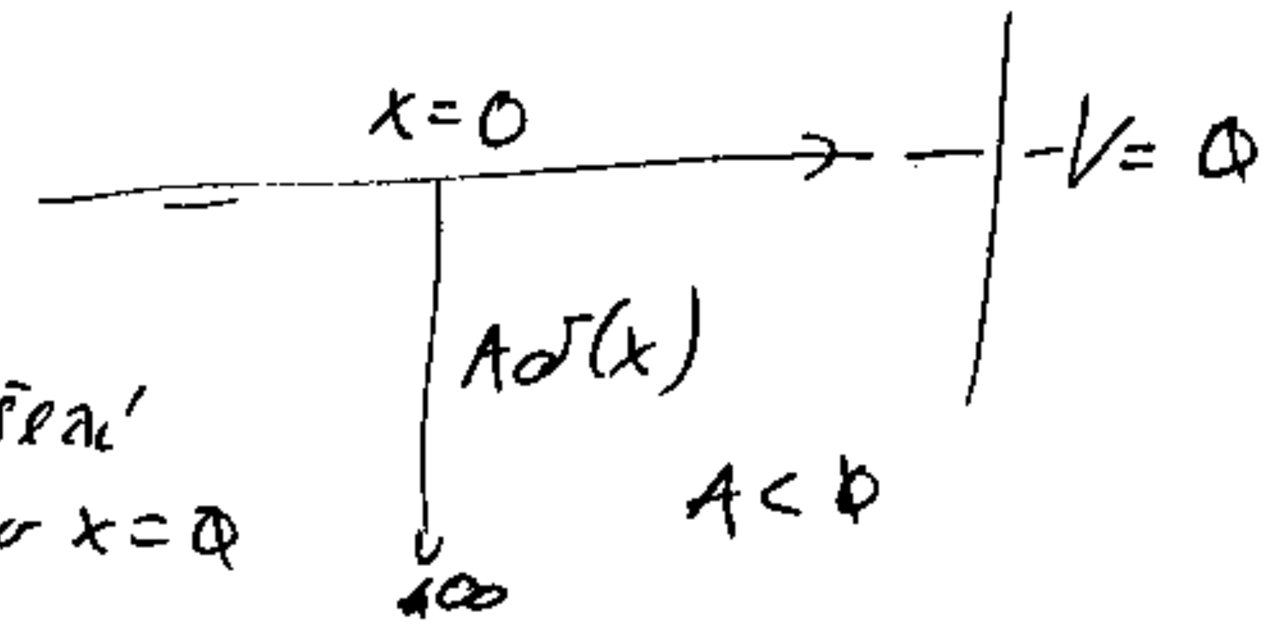
$\delta(x)$ potenciál, vázaný stav

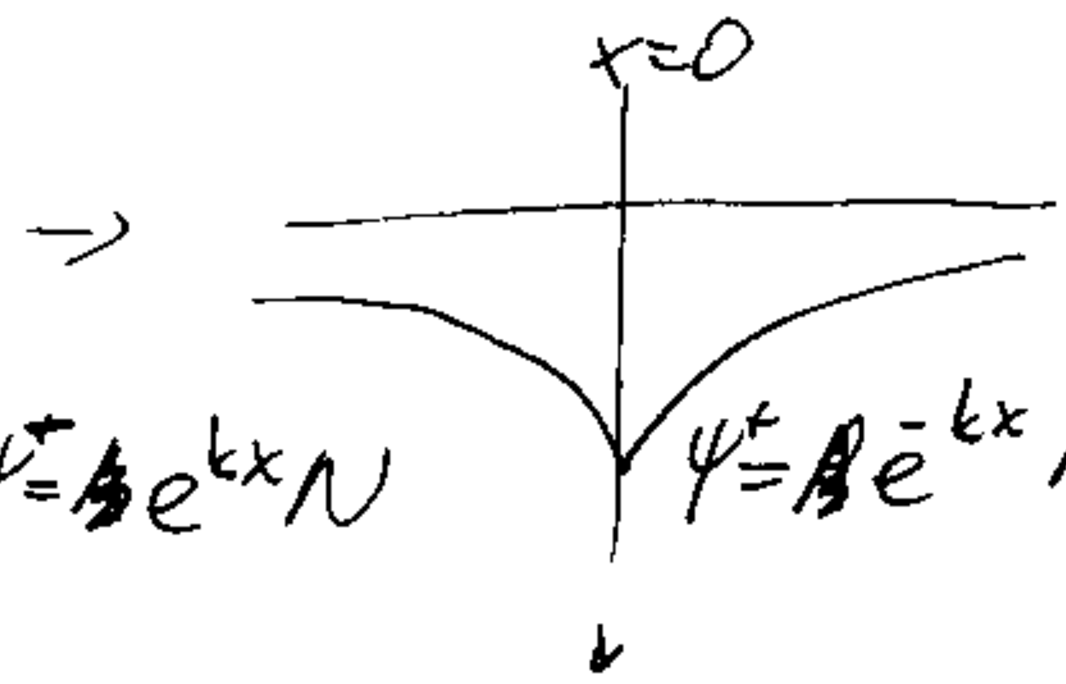
$$\left(\frac{p^2}{2m} + A\delta(x)\right)\psi(x) = E\,\psi(x)$$



- vázaný stav $\rightarrow E < 0 \rightarrow e^{\pm kx}$ řešení mimo $x=0$
- v $x=0$ $\quad \frac{p^2}{2m}\psi = -A\delta(x)\psi(x)$

$$+\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = +A\delta(x)\psi(x)$$

– nekonečná 2. derivace
– skok v 1. derivaci
ψ stále musí být spojitá



$\psi^- = N e^{kx}$ $\qquad \psi^+ = N e^{-kx}$

$\rightarrow \psi^-(0) = \psi^+(0)$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + A\delta(x)\psi(x) = E\,\psi(x)$$

"$\delta(x)$ má smysl při integraci"

$$-\int_{-\varepsilon}^{\varepsilon}\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \int_{-\varepsilon}^{\varepsilon} A\delta(x)\psi(x) = \int_{-\varepsilon}^{\varepsilon} E\,\psi(x) \qquad \varepsilon \rightarrow 0$$

$$-\frac{\hbar^2}{2m}\left[\frac{\partial\psi}{\partial x}\right]_{-\varepsilon}^{\varepsilon} + A\psi(0) \qquad \underset{\varepsilon E\psi(0)}{\rightarrow 0}$$

$$\rightarrow \frac{\hbar^2}{2m}\left[\psi_+^{(1)}(0) - \psi_-'(0)\right] = A\psi(0)$$

$$\frac{\hbar^2}{2m}\left[N(-k)e^{-kx} - Nk\,e^{kx}\right] = AN\,e^0 \qquad A<0$$

$$-\frac{2k\hbar^2}{2m} = A$$

$$\rightarrow k = -\frac{Am}{\hbar^2} = \frac{|A|m}{\hbar^2}$$

Normalizace

$$N^2\int_0^\infty e^{-\frac{2|A|m}{\hbar^2}x} = \frac{N^2\hbar^2}{2|A|m}\left[e^{-\frac{2|A|m}{\hbar^2}x}\right]_{0\rightarrow 1}^{\infty\rightarrow 0} = \frac{N^2\hbar^2}{2|A|m} = \frac{1}{2}$$

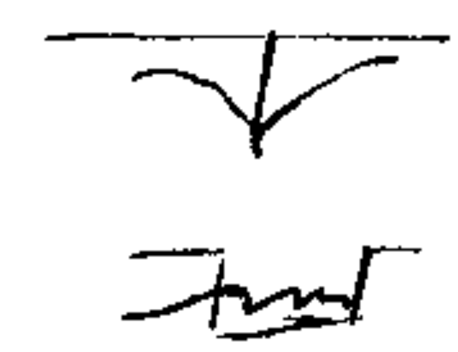
jedna polovina intervalu

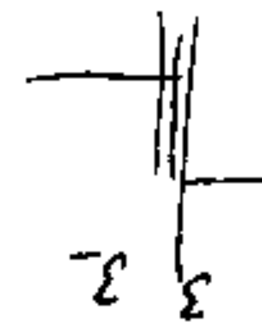
$$\Rightarrow N = \frac{\sqrt{|A|m}}{\hbar}$$

$$\rightarrow \psi = \frac{\sqrt{|A|m}}{\hbar}e^{-k|x|} \qquad k = \frac{|A|m}{\hbar^2}$$

- větší hmotnost $\rightarrow$ užší ψ ✓
- menší $|A|$ $\rightarrow$ širší ψ ✓ $\Leftrightarrow$ menší masa ✓

$$-\int_{-\varepsilon}^{+\varepsilon}\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+\int_{-\varepsilon}^{\varepsilon}V(x)\psi(x)=\int_{-\varepsilon}^{\varepsilon}E\,\psi(x) \qquad \varepsilon\to 0$$





$\to \varepsilon V^+\psi_+ \varepsilon V^-\psi$

$\to 0$ pro $\varepsilon\to 0$ $\to$ hladká $\psi(x)$

$\delta(\phi-\pi)$ potenciál pro periodickou částici / rotor

$\psi_n=\frac{1}{\sqrt{2\pi}}e^{in\phi}$, $n\in\mathbb{Z}$, $\phi\in(0,2\pi)$

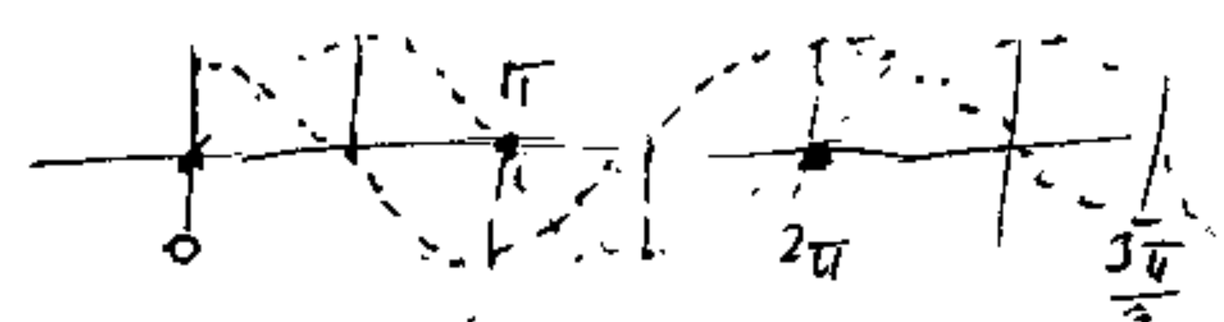
maticové elementy pro $V=A\,\delta(\phi-\pi)$?

$$\langle n|V|m\rangle=\frac{1}{2\pi}\int_0^{2\pi}e^{-in\phi}A\,\delta(\phi-\pi)\,e^{im\phi}\,d\phi$$

$$=\frac{A}{2\pi}\int_0^{2\pi}\delta(\phi-\pi)\,e^{i(m-n)\phi}\,d\phi$$

$$=\frac{A}{2\pi}\int_0^{2\pi}\delta(\phi-\pi)\left[\cos[(m-n)\phi]+i\sin[(m-n)\phi]\right]d\phi$$

$$=\frac{A}{2\pi}\left[\cos((m-n)\pi)+i\sin[(m-n)\pi]\right]$$



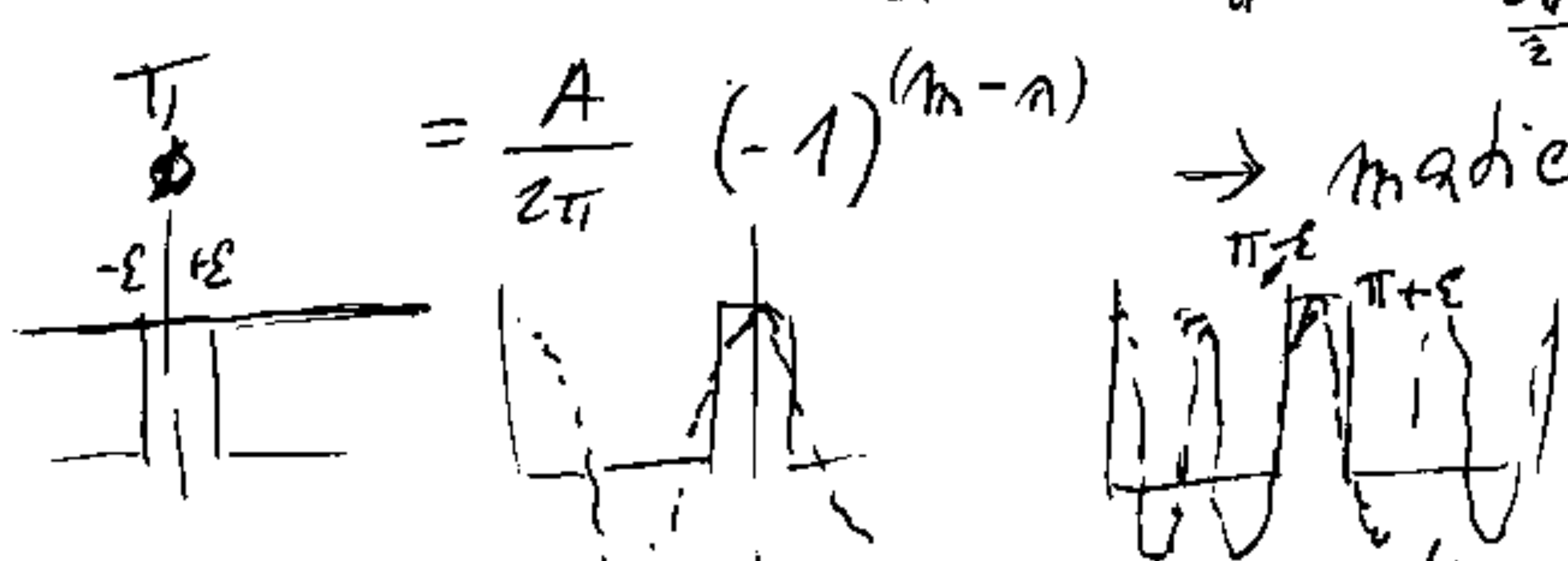
$= 0$

$$=\frac{A}{2\pi}(-1)^{(m-n)} \to \text{matice plná}$$

$$\begin{matrix} & & \vdots & & \\ \cdots & 1 & -1 & 1 & \cdots \\ \cdots & -1 & 1 & -1 & \cdots \\ \cdots & 1 & -1 & 1 & \cdots \\ & & \vdots & & \end{matrix}$$

$\to$ přispívají všechny ϕ_n k ψ_0, ψ_0 má cusp

c.f.



$m-n=0$ $\quad$ $m-n=2$ $\quad$ $m-n=4$

$$\int_{\pi-\varepsilon}^{\pi+\varepsilon}\cos[(m-n)\phi]\,d\phi = -\frac{\sin[(m-n)\phi]}{(m-n)}\Bigg|_{\pi-\varepsilon}^{\pi+\varepsilon}$$

$m\neq n$

$\to$ maticové elementy se snižují pro $|(m-n)|$ rostoucí

$\to$ není cusp

2 state vývoj čas

- máme systém s 2. degenerovanými hladinami

$$H_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \phi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \phi_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

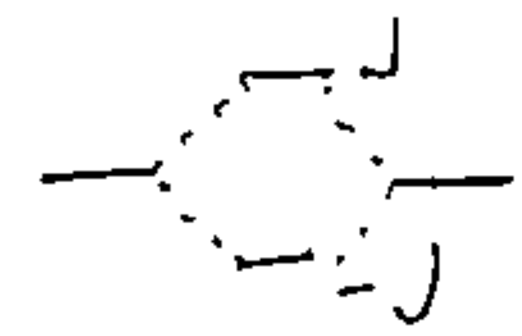
- ~~poruchy (el. pole, mag. pole...)~~ $\cancel{V = \begin{pmatrix} 0 & J \\ J & 0 \end{pmatrix}}$

$\cancel{H_0 + V = \begin{pmatrix} 0 & J \\ J & 0 \end{pmatrix}}$

- operátor $A = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$
- porucha $V = \begin{pmatrix} 0 & J \\ J & 0 \end{pmatrix}$ (el. pole, mag. pole, ...)
- Nové vlastní energie a stavy?

$$H_0 + V = \begin{pmatrix} 0 & J \\ J & 0 \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & J \\ J & -\lambda \end{vmatrix} = \lambda^2 - J^2 = (\lambda - J)(\lambda + J) = 0 \to \lambda = \pm J$$



$$\lambda = J \quad \begin{pmatrix} -J & J \\ J & -J \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0 \to \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = |+\rangle$$

$$\lambda = -J \quad \begin{pmatrix} J & J \\ J & J \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0 \to \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = |-\rangle$$

nezávisí na velikosti J & pouze ze "symetrie"

- časový vývoj obecně: $\Psi(t) = \sum_n c_n e^{-iE_n t/\hbar} \phi_n$

$$\Psi(t) = c_+ e^{-iJt/\hbar}|+\rangle + c_- e^{iJt/\hbar}|-\rangle$$

- pokud $\Psi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \phi_0$ jaký je časový vývoj Ψ

$\to$ potřebujeme c_+ a c_-

$$c_+ = \langle + | \Psi_0 \rangle = \frac{1}{\sqrt{2}}(1 \;\; 1)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}$$

$$c_- = \langle - | \Psi_0 \rangle = \frac{1}{\sqrt{2}}(1 \;\; -1)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}$$

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1 \checkmark$$

$$\Psi(t) = \frac{1}{\sqrt{2}} e^{-iJt/\hbar} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2} e^{iJt/\hbar}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} e^{-iJt/\hbar} + e^{iJt/\hbar} \\ e^{-iJt/\hbar} - e^{+iJt/\hbar} \end{pmatrix}$$

$$= \begin{pmatrix} \cos(Jt/\hbar) \\ i\sin(Jt/\hbar) \end{pmatrix}$$

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- operátor $A = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$ v bázi ϕ_0, ϕ_1
- jaký je čas. vývoj v bázi $\{\phi_0, \phi_1\}$ a $\{|+\rangle, |-\rangle\}$

$$\psi(t) = \begin{pmatrix} \cos(Jt/\hbar) \\ i\sin(Jt/\hbar) \end{pmatrix}$$

$$\langle A \rangle = \left(\cos(Jt/\hbar) \quad -i\sin(Jt/\hbar)\right) \begin{pmatrix} A & B \\ B & A \end{pmatrix} \begin{pmatrix} \cos(Jt/\hbar) \\ +i\sin(Jt/\hbar) \end{pmatrix}$$

$$= (\cos \quad -i\sin) \begin{pmatrix} A\cos(Jt/\hbar) + iB\sin(Jt/\hbar) \\ B\cos(Jt/\hbar) + iA\sin(Jt/\hbar) \end{pmatrix}$$

$$= A\cos^2(Jt/\hbar) + \cancel{iB\cos(\ )\sin(\ )} - \cancel{iB\cos(\ )\sin(\ )} + A\sin^2(\ )$$

$$= A$$

$$U^+AU = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} A & B \\ B & A \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} A+B & A-B \\ A+B & B-A \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} A+B+A+B & A-\cancel{B}+(\cancel{B}-A) \\ A+B-(A+B) & A-B-(B-A) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2A+2B & 0 \\ 0 & 2A-2B \end{pmatrix} = \begin{pmatrix} A+B & 0 \\ 0 & A-B \end{pmatrix}$$

$$\psi = \frac{1}{\sqrt{2}} |+\rangle e^{-iJt/\hbar} + \frac{1}{\sqrt{2}} |-\rangle e^{iJt/\hbar}$$

$$\langle A \rangle = \frac{1}{2} \left(\langle +| e^{iJt/\hbar} + \langle -| e^{-iJt/\hbar} \right) \left(|+\rangle (A+B) \langle +| + |-\rangle (A-B) \langle -| \right) \left(|+\rangle e^{-iJt/\hbar} + |-\rangle e^{-iJt/\hbar} \right)$$

$$= \frac{1}{2} (A+B+A-B) = A$$